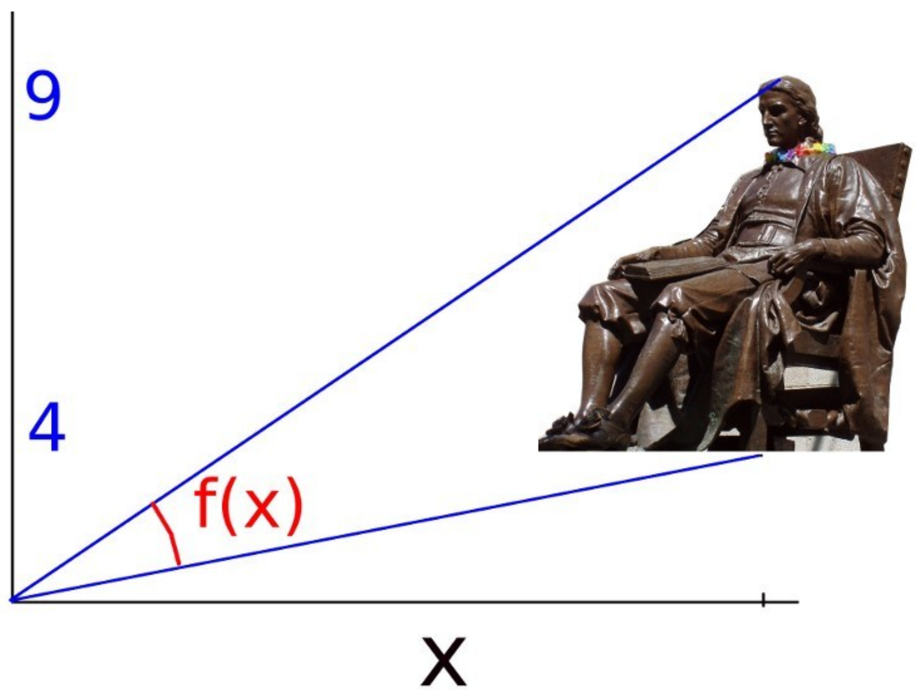
INTRODUCTION TO CALCULUS

MATH 1A

Unit 14: Worksheet

Problem 1: You are a tourist looking at the John Harvard Statue. If you are too close below it, the viewing angle becomes small. If you are far away, the viewing angle decreases again. There is an optimal distance where the viewing angle is maximal?



At which distance x do you see most of the John Harvard Statue? Assume the part you want to see 4 to 9 feet higher than your eyes.

Problem 1: Verify that the angle you see from the statue is

$$f(x) = \arctan(\frac{9}{x}) - \arctan(\frac{4}{x}) \ .$$

Solution:
By definition of the arctan, the larger angle is $\arctan(9/x)$. The smaller angle is $\arctan(4/x)$. The difference is $f(x)$.

Problem 2: Differentiate $f(x)$ to find the minimum.

Solution:
This is tougher as we do not have the chain rule yet. One way is to write $\arctan(4/x) = \text{arccot}(x/4)$ and using the derivative of $\text{arccot}(x)$. The derivative simplifies to $5(36 - x^2)/((16 + x^2)(81 + x^2))$ which is zero at $x = 6$.

Problem 3: Are there any boundary points or points where f is not differentiable?

Solution:
As written, there is a singularity at $x = 0$. However, we could actually assign to the point a nice value and even continue through 0 to have a differentiable function but then the distance for negative x would be negative. As written, the function is not differentiable at $x = 0$.

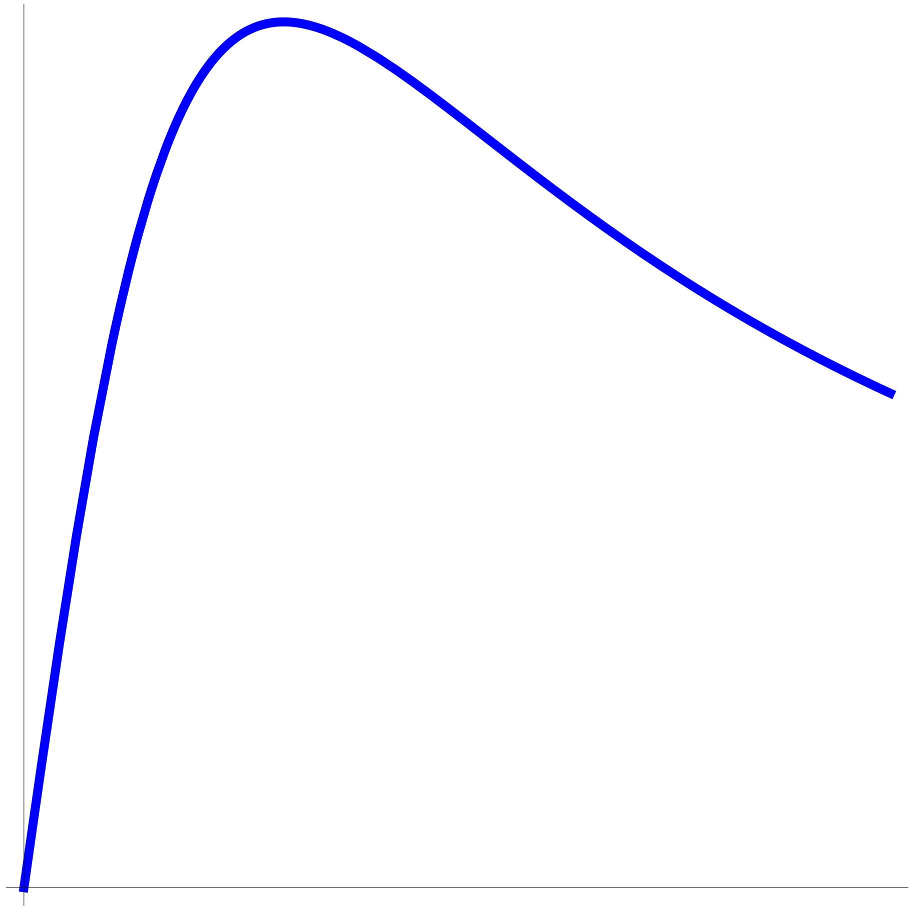
Problem 4: Is there a global maximum of f? If yes, where is it?

Solution:
The global maximum is 6.

Problem 5: Is there a global minimum of f? If yes, where is it?

Solution:
There is a global minimum $x = 0$.

Here is a graph of part of the function f.



Oliver Knill, knill@math.harvard.edu, Math 1a, Spring, 2024