

# INTRODUCTION TO CALCULUS

MATH 1A

## UNIT 18: WORKSHEET

**17.1. Problem 1:** A couple of weeks ago was **Valentine day**. The heart function  $(x^2 + y^2 - 1)^3 - x^2y^3 = 0$  relates  $x$  with  $y$ , but we can not write the curve as a graph of a function  $y = y(x)$ . Extracting  $y$  or  $x$  is difficult. We still can find the derivative  $y'$  knowing  $x = 1, y = 1$ .

### Solution:

Using the chain rule, we can take the derivative

$$3(x^2 + y^2 - 1)(2x + 2yy') - 2xy^3 - x^2 3y^2 y' = 0$$

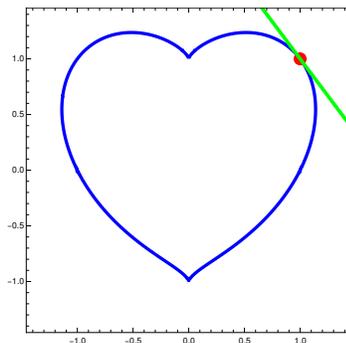
We can now solve solve for  $y'$

$$y'(x) = -\frac{3(x^2 + y^2 - 1)2x - 2xy^3}{3(x^2 + y^2 - 1)2y - 3x^2y^2}.$$

Filling in  $x = 1, y = 1$  gives  $y' = -4/3$ . We have computed the slope of  $g$  without knowing  $g$ . Magic! It comes even better as we have seen in class: we can already after taking the derivative plug in  $x=1$  and  $y=1$ . And get

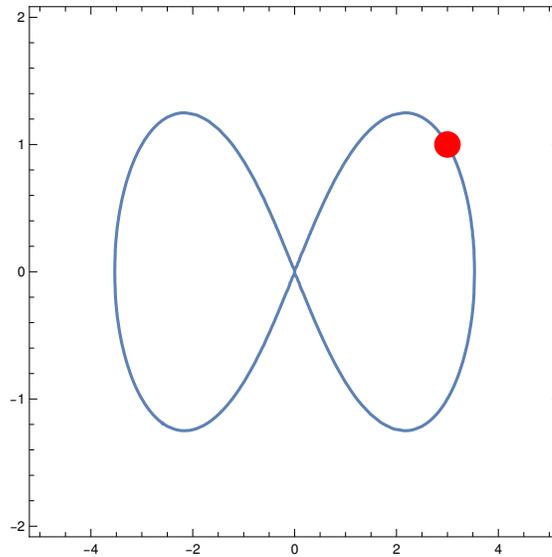
$$3(1 + 1 - 1)(2 + 2y') - 2 - 3y' = 0$$

from which we can solve for  $y' = -4/3$  faster without all the gymnastics.



**Problem 2:** The lemniscate curve is given by  $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ . Find the slope of the tangent at  $(3, 1)$ . This is the derivative of  $y'$  at  $x = 3$ .

Single Variable Calculus



**Solution:**

Take the derivative  $4(x^2 + y^2)(2x + 2yy') = 50x - 50yy'$ . Now plug in  $x = 3, y = 1$  to get

$$y' = -9/13.$$