UNIT 26: WORKSHEET

Some True-False Problems

1) T F The fundamental theorem of calculus assures that $\int_a^b f'(x)\ dx = f(a) - f(b)$.

Solution:
The order is reversed on the right hand side.

2) T F If $\int_0^x f(t)\ dt$ is monotonically increasing in x for $0 \le x \le 1$, then $f(x) \ge 0$ on $0 \le x \le 1$.

Solution:
The derivative of the integral is $f(x)$ and because the integral is monotonically increasing, this derivative is positive or zero.

3) T F For any continuous function f, the integral $\int_a^b f(x)\ dx$ is the area under a curve and therefore always positive or zero.

Solution:
It can be a signed area and negative. The statement is false

4) T F If $f_c(x)$ has a minimum x_c which is present for $c < 0$ and disappears for $c > 0$, then $c = 0$ is a catastrophe.

Solution:
This is a definition.

A Catastrophe problem

Problem) Catastrophes (10 points)

Consider the family of functions $f(x) = x^3 + cx$ on the real line.
a) (5 points) Find all critical points of f, depending on c.
b) (2 points) Using the second derivative test, determine which are minima and which are maxima.
b) (3 points) For which value of c does a catastrophe occur?

Solution:
a) The critical points are $3x^2 + c = 0$ which means $x = \sqrt{-c/3}$. There are no critical points for $c > 0$ and two different critical points for $c < 0$.
b) The second derivative is $6x$ which is negative for $x < 0$ and positive for $x > 0$. The point $\sqrt{-c/3}$ is a local minimum for $c < 0$. The point $-\sqrt{-c/3}$ is a local maximum for $c < 0$.
c) The catastrophe appears at the parameter $c = 0$ because a local minimum, present for $c < 0$ disappears for $c \geq 0$.

Problem) Integrals (10 points)

Two of the following 6 integrals can be solved (with the methods we know) two more are harder to guess at this stage. Two can not be solved.
a) $\int \ln(x) \, dx$
b) $\int 1/\ln(x) \, dx$
c) $\int \ln(x)/x \, dx$
d) $\int 1/(\ln(x)x) \, dx$
e) $\int x/\ln(x) \, dx$
f) $\int x \ln(x) \, dx$

Solution:
a) $x \ln(x) - x + C$. (will be solve later)
b) Can not be solved
c) $\ln(x)^2/2 + C$
d) $\ln(\ln(x)) + C$
e) Can not be solved
f) $-x^2/4 + x^2 \ln(x)/2 + C$ (will be solved later)