UNIT 27: WORKSHEET

The sigmoid function

Problem 1: In some texts, you see the sigmoid function written as $\sigma(x) = 1/(1 + e^{-x})$. Verify that this agrees with $\sigma(x) = e^x/(1 + e^x)$.

Solution:
Just multiply nominator and denominator by e^x.

The fundamental theorem

$$\int_0^x f'(t)\, dt = f(x) - f(0), \qquad \frac{d}{dx}\int_0^x f(t)\, dt = f(x)\,.$$

Problem 2: Write down both expressions in the case $f(x) = \cos(x)$.

Solution:
a) $\int_0^x -\sin(t)\, dt = \cos(t)|_0^x = \cos(x) - \cos(0)$. b) $\frac{d}{dx}\int_0^x \cos(t)\, dt = \frac{d}{dx}[\sin(x) - \sin(0)] = \cos(x)$.

The discrete fundamental theorem

Write $Sf(x) = f(0) + f(1) + \cdots + f(x-1)$. and $Df(x) = f(x+1) - f(x)$.

Problem 3: Verify that $DSf(x) = f(x)$. This is the analog of $\frac{d}{dx} \int f(t)\, dt = f(x)$.

Solution:
$DSf(x) = [f(0) + f(1) + \cdots + f(x-1) + f(x)] - [f(0) + f(1) + \cdots + f(x-1)] = f(x)$.

Problem 4: Verify that $SDf(x) = f(x) - f(0)$. This is the analog of $\int_0^x f'(t)\, dt = f(x) - f(0)$.

Solution:
$SDf(x) = [f(1) - f(0)] + [f(2) - f(1)] + \ldots + [f(x) - f(x-1)] = f(x) - f(0)$.