

INTRODUCTION TO CALCULUS

MATH 1A

UNIT 27: WORKSHEET

The sigmoid function

Problem 1: In some texts, you see the sigmoid function written as $\sigma(x) = 1/(1 + e^{-x})$. Verify that this agrees with $\sigma(x) = e^x/(1 + e^x)$.

The fundamental theorem

$$\int_0^x f'(t) dt = f(x) - f(0), \quad \frac{d}{dx} \int_0^x f(t) dt = f(x).$$

Problem 2: Write down both expressions in the case $f(x) = \cos(x)$.

The discrete fundamental theorem

Write $Sf(x) = f(0) + f(1) + \cdots + f(x-1)$. and $Df(x) = f(x+1) - f(x)$.

Problem 3: Verify that $DSf(x) = f(x)$. This is the analog of $\frac{d}{dx} \int f(t) dt = f(x)$.

Problem 4: Verify that $SDf(x) = f(x) - f(0)$. This is the analog of $\int_0^x f'(t) dt = f(x) - f(0)$.

Single Variable Calculus

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