INTRODUCTION TO CALCULUS

MATH 1A

UNIT 32: WORKSHEET

Problem 1: Verify that the function $f(x) = \frac{1}{x}$ is a probability density function on $[1, e]$. This means $\int_1^e f(x)\, dx = 1$ and that f is non-negative.

Solution:
$F(x) = \ln(x)$ on $[1, e]$. Now $F(e) = 1$ and $F(1) = 0$. This is a CDF and f is a PDF.

Problem 2: Find the expectation

$$m = \int_1^e x f(x)\, dx$$

of this distribution function f.

Solution:
$\int_1^e x 1/x\, dx = e - 1$.

Problem 3: Find the variance

$$\int_1^e x^2 f(x)\, dx - m^2$$

of f.

Solution:
$\int_1^e x^2/x \; dx = (e^2 - 1)/2$. The variance is $(e^2 - 1)/2 - (e - 1)^2$.