# Lecture 33: Worksheet

This worksheet as well as the solutions was generated by Sofia, a bot written in the academic year 2003/2004 using grant from the Harvard Provost together with Harvard students **Johnny Carlsson**, **Andrew Chi** and **Mark Lezama**. At that time, people have laughed at the chat bot idea. In the means time first came players like Google, Siri, Cortana, Wolfram alpha. Today, worksheets can be generated with more sophisticated transformer models like ChatGPT,Llama or Gemini who went through the attention revolution in 2017. We had in class ChatGPT 4 generate a worksheet including graphics. They are not yet as pretty as the output of this worksheet generator, done 21 years ago.

**1** Differentiate the following functions:
a) $f(x) = 3x\cos(x)$
b) $f(x) = 8$
c) $f(x) = x - \sin(x)$

**Solution:**
a) $f'(x) = 3(\cos(x) - x\sin(x))$
b) $f'(x) = 0$
c) $f'(x) = 1 - \cos(x)$

**2** Integrate the following functions:
a) $f(x) = \frac{\log(x) - 1}{\log^2(x)}$
b) $f(x) = -2(x\cot(x) - 1)\csc(x)$
c) $f(x) = 3\left(\frac{1}{x^2} + 1\right)$

**Solution:**
a) $\int f(x) = \frac{x}{\log(x)} + C$
b) $\int f(x) = 2x\csc(x) + C$
c) $\int f(x) = 3\left(x - \frac{1}{x}\right) + C$

**3** Differentiate the following functions:
a) $f(x) = 4\left(\frac{1}{x^2} - \sin(x)\right)$
b) $f(x) = 12\sin(x)$
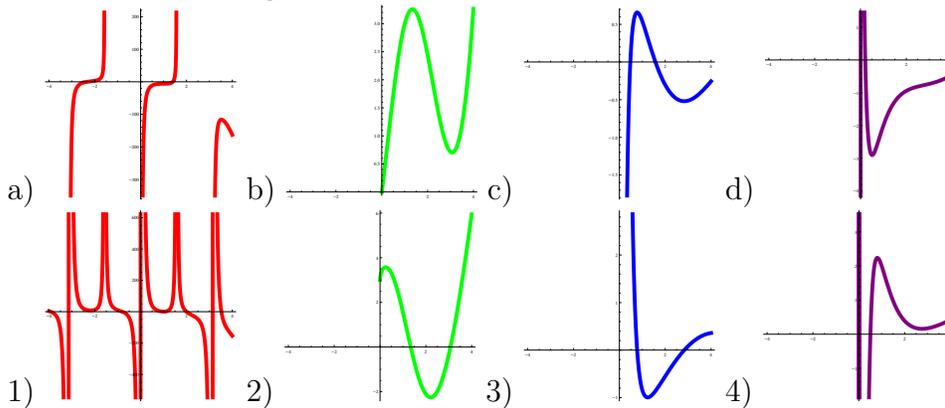c) $f(x) = \frac{x + \frac{1}{x}}{x^4}$

**4** Integrate the following functions:
a) $f(x) = \frac{4\log^3(x)}{x}$
b) $f(x) = 2\left(e^x + \cos(x)\right)$
c) $f(x) = \frac{2\left(\cot(x) - 2x\csc^2(x)\right)}{\sqrt{x}}$

**5** Match the following functions with derivatives:

**6** Find the critical points of the following functions:
a) $f(x) = x^2 - 13x + 36$
b) $f(x) = x^3 - 17x^2 + 92x - 160$
c) $f(x) = x^2 - 12x + 27$

**Solution:**

a) $f'(x) = \left\{\left\{x \to \frac{11}{2}\right\}\right\}$

b) $f'(x) = \left\{\left\{x \to \frac{1}{3}\left(22 - \sqrt{13}\right)\right\}, \left\{x \to \frac{1}{3}\left(22 + \sqrt{13}\right)\right\}\right\}$

c) $f'(x) = \left\{\{x \to 6\}, \left\{x \to \frac{22}{3}\right\}\right\}$