

2/28/2024: First Hourly Practice B

**”By signing, I affirm my awareness of the standards of the
Harvard College Honor Code.”**

Your Name:

Please write neatly. Use the same page for the answer if possible.

1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		100

Problem 1) TF questions (10 points) No justifications are needed.

- 1) T F The function $f(x) = \exp(-x^2) - 1$ has the root $x = 0$.

- 2) T F $\ln(\ln(e)) = 0$.

- 3) T F The function $f(x) = x^2/(1-x^2)$ is continuous everywhere on the real axes.

- 4) T F $f'(x) = 0$ and $f'''(0) < 0$ at $x = 0$ assures that f has a maximum at $x = 0$.

- 5) T F A continuous function on $(-1, 1)$ has a global maximum and a global minimum.

- 6) T F If f and g are differentiable then f/g is differentiable

- 7) T F The Groundhog function $\arctan(\tan(x))$ is continuous everywhere

- 8) T F $\lim_{x \rightarrow 0} x^x = 0$.

- 9) T F $\frac{d}{dx} e^x / x = \frac{e^x x - e^x}{x^2}$.

- 10) T F The derivative of $h(x) = f(x) * g(x)$ is $h'(x) = f'(x)g'(x)$.

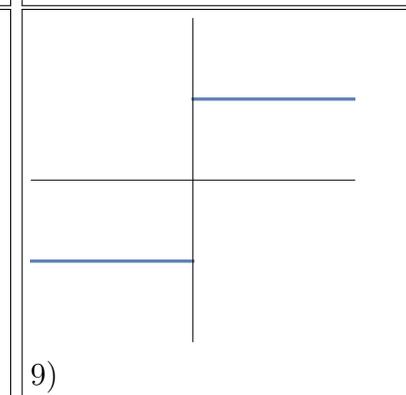
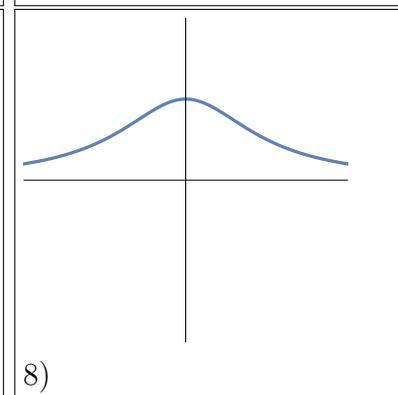
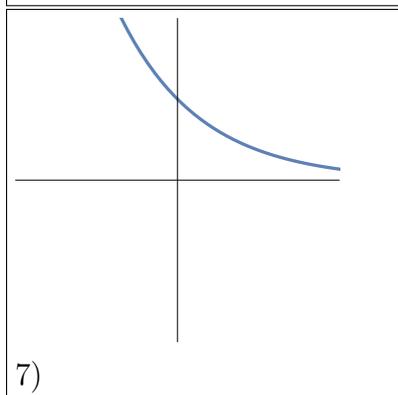
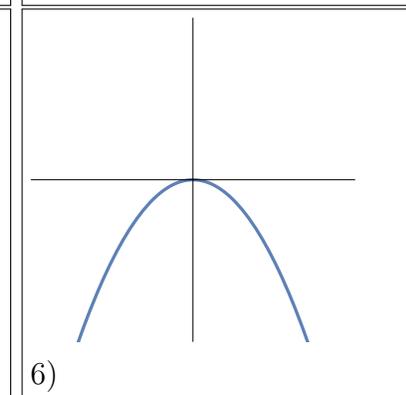
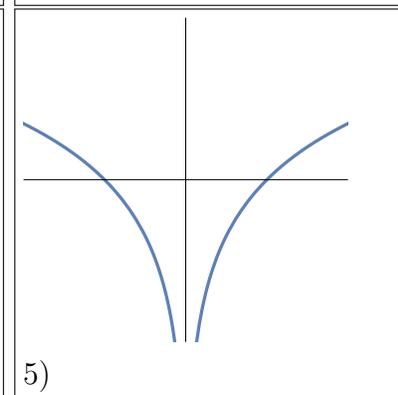
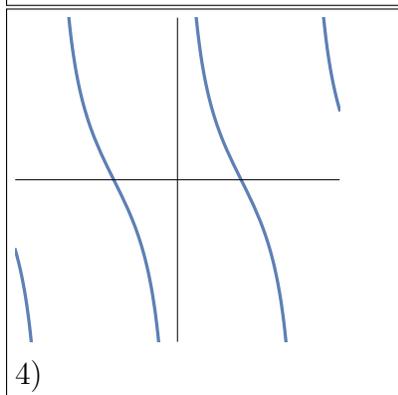
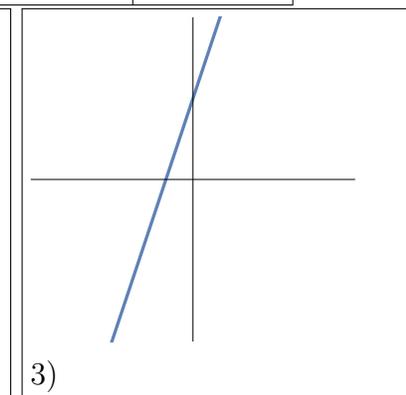
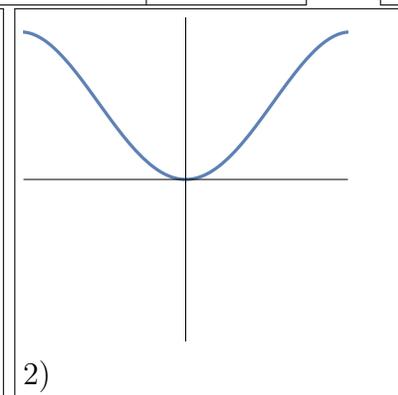
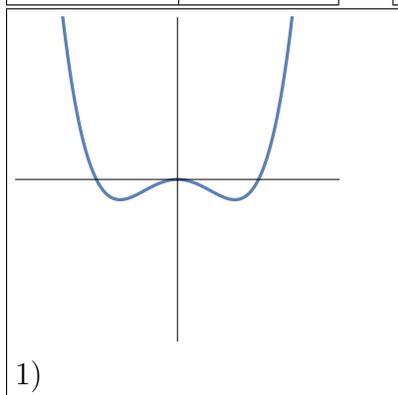
Problem 2) Analysis (10 points) No justifications are needed.

Match the functions with the graphs.

Function	Enter 1-9
$1/(1+x^2)$	
$\cot(2x)$	
$3x+1$	

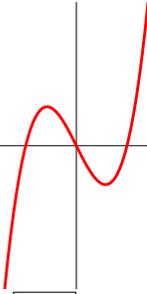
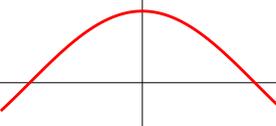
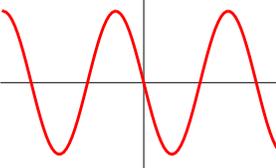
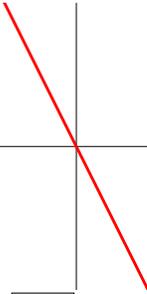
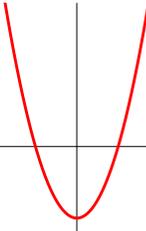
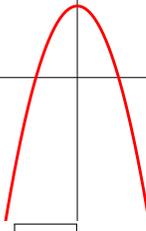
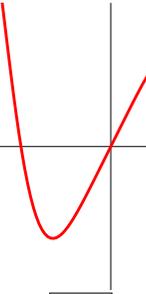
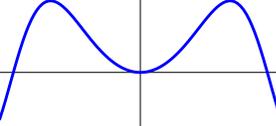
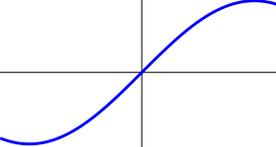
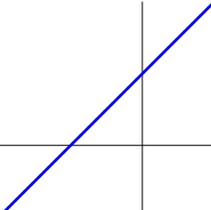
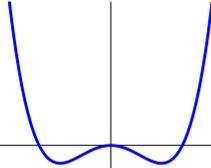
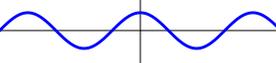
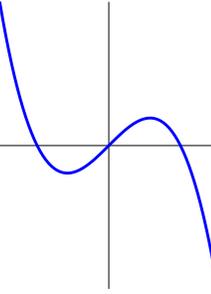
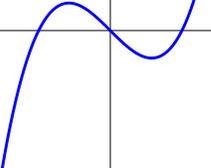
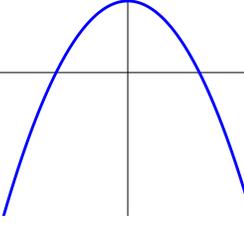
Function	Enter 1-9
$x \sin(x)$	
$\exp(-x)$	
$\ln(x)$	

Function	Enter 1-9
$\text{sign}(x)$	
$x^4 - x^2$	
$-x^2$	



Problem 3) Graphing (10 points) No justifications are needed.

In the first pictures, we see the first derivatives f' . Match them with the functions f in 1-8. Note that the functions above are the derivative and the functions below are the functions.

 <p>a) → <input type="checkbox"/></p>	 <p>b) → <input type="checkbox"/></p>	 <p>c) → <input type="checkbox"/></p>	 <p>d) → <input type="checkbox"/></p>
 <p>e) → <input type="checkbox"/></p>	 <p>f) → <input type="checkbox"/></p>	 <p>g) → <input type="checkbox"/></p>	 <p>h) → <input type="checkbox"/></p>
 <p>1)</p>	 <p>2)</p>	 <p>3)</p>	 <p>4)</p>
 <p>5)</p>	 <p>6)</p>	 <p>7)</p>	 <p>8)</p>

Problem 4) Continuity (10 points)

Each of the following functions has a point x_0 , where the function is not defined. Find the limit $\lim_{x \rightarrow x_0} f(x)$ or state that the limit does not exist.

a) (2 points) $f(x) = \frac{1-2x^3}{1-x}$, at $x_0 = 1$.

b) (2 points) $f(x) = \sin(\sin(5x))/\sin(7x)$, at $x_0 = 0$.

c) (2 points) $f(x) = \frac{\exp(-3x)-1}{\exp(2x)-1}$, at $x_0 = 0$.

d) (2 points) $f(x) = \frac{2x}{\ln|x|}$, at $x_0 = 0$.

e) (2 points) $f(x) = \frac{(x-1)^{10}}{(x+1)^{10}}$, at $x_0 = -1$.

(*) As usual, we extend continuity to functions for which a continuation is possible to initially not defined points, like $f(x) = (x^2 - 1)/(x - 1)$, which is considered continuous everywhere because we can fill in a function value for $x = 1$ which makes it continuous.

Problem 5) Derivatives (10 points)

Find the derivatives of the following functions. If you use a differentiation rule, note which one you use.

a) (2 points) $f(x) = \sqrt{\ln(x+1)}$.

b) (3 points) $f(x) = 7 \sin(x^3) + \frac{\ln(5x)}{x}$.

c) (3 points) $f(x) = \ln(\sqrt{x}) + \arctan(x^3)$.

d) (2 points) $f(x) = e^{5\sqrt{x}} + \tan(x)$.

Problem 6) Limits (10 points)

Find the limits $\lim_{x \rightarrow 0} f(x)$ for the following functions:

a) (2 points) $f(x) = \frac{\exp(3x) - \exp(-3x)}{\exp(5x) - \exp(-5x)}$.

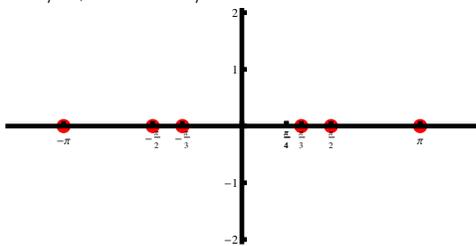
b) (3 points) $f(x) = \frac{\cos(3x) - 1}{\sin^2(x)}$.

c) (3 points) $f(x) = [\arctan(x) - \arctan(0)]/x$.

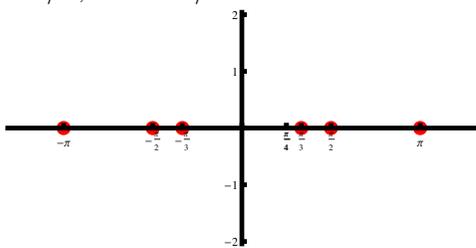
d) (2 points) $f(x) = \frac{\ln(7x)}{\ln(11x)}$.

Problem 7) Functions (10 points) no explanations needed

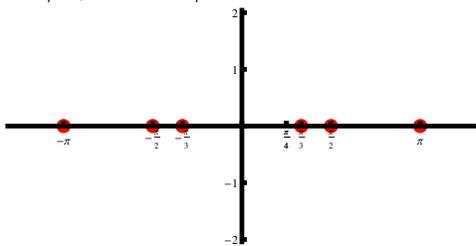
a) Draw the sin function and mark the values of $\sin(x)$ at $x = 0, x = \pm\pi/4, x = \pm\pi/3, x = \pm\pi/2$.



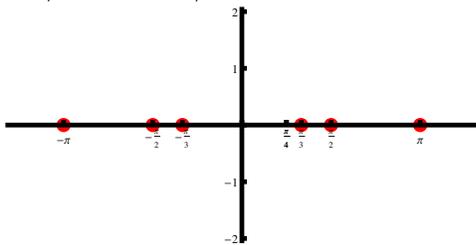
b) Draw the cos function and mark the values of $\cos(x)$ at $x = 0, x = \pm\pi/4, x = \pm\pi/3, x = \pm\pi/2$.



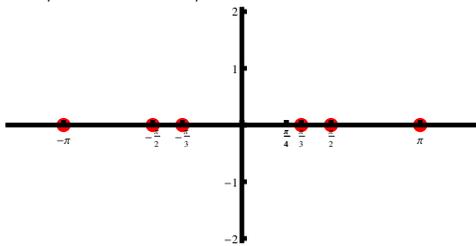
c) Draw the tan function and mark the values of $\tan(x)$ at $x = 0, x = \pm\pi/4, x = \pm\pi/3, x = \pm\pi/2$.



d) Draw the cot function and mark the values of $\cot(x)$ at $x = 0, x = \pm\pi/4, x = \pm\pi/3, x = \pm\pi/2$.



e) Draw the sinc function $f(x) = \sin(x)/x$ and mark the points $x = 0, x = \pm\pi/4, x = \pm\pi/3, x = \pm\pi/2$.



Problem 8) Extrema (10 points)

You got a batch of strong **Neodym magnets**. They are ring shaped. Assume the inner radius is x , the outer radius y is 1 and the height is $h = x$, we want to maximize the surface area $A = 2\pi(y-x)h + 2\pi(y^2 - x^2)$. This amount of maximizing

$$f(x) = 2\pi(1-x)x + 2\pi(1-x^2)$$



- (2 points) Using that $f(x)$ is a surface area, on what interval $[a, b]$ needs f to be considered?
- (3 points) Find the local maxima of f inside the interval.
- (3 points) Use the second derivative test to verify it is a maximum.
- (2 points) Find the global maximum on the interval.

Problem 9) Algebra (10 points)

Simplify the following terms. \log denotes the natural log and \log_{10} the log to the base 10. Each result in a)-c) is an integer or a fraction

a) (2 points) $\exp(\log(2)) + e^{3\log(2)}$

b) (2 points) $\log(1/e) + \exp(\log(2)) + \log(\exp(3))$.

c) (2 points) $\log_{10}(1/100) + \log_{10}(10000)$

d) (4 points) Simplify

$$\cos(\arccos(x)) .$$

to get the derivative of $\cos(\arccos(x))$.

Problem 10) Linearization (10 points, 5 points each)

a) (5 points) Use linearization to estimate $\sqrt[5]{1.001}$.

b) (5 points) Use linearization to estimate $e^{\ln(2)+0.01}$.