4/3/2024: Second Hourly Practice A

"By signing, I affirm my awareness of the standards of the Harvard College Honor Code."

Your Name:

Please write neatly. Use the same page for the answer if possible.

1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		100

Problem 1) TF questions (10 points) No justifications are needed.

1) T F Differentiating $f(f^{-1}(x)) = x$ allows to get $\frac{d}{dx} f^{-1}(x)$.

2) T F $\int_0^1 x^3 \, dx = \lim_{n \to \infty} \sum_{k=0}^{n-1} (\frac{k}{n})^3 \frac{1}{n}$.

3) T F If $0 \le f(x) \le 1$, then $0 \le \int_0^1 f(x) \, dx \le 1$

4) T F If $f(x)$ is differentiable integral $\int_0^1 f(x) \, dx$ can be approximated by Riemann sums.

5) T F If $f(x) = 1$ everywhere, then $\int_a^b f(x) \, dx$ is the length of the interval $[a, b]$.

6) T F If f is continuous, then $\int_a^b -f(x) \, dx = -\int_a^b f(x) \, dx$.

7) T F The fundamental theorem of calculus implies $\int_a^b f''(x) \, dx = f'(b) - f'(a)$ if f'' is a differentiable function.

8) T F The family $f_c(x) = x^2 + c$ experiences a catastrophe at $c = 0$.

9) T F If f is differentiable and T is a Newton step, then $T(x), T^2(x) \ldots$ converges to a root of x.

10) T F If f is continuous, then $\int_a^b f(-x) \, dx = -\int_a^b f(x) \, dx$.

Problem 2) Theorems (10 points) No justifications are needed.

Fill in the missing part into the empty box to make a true statement.

a) $\int_0^1 f'(x)\,dx =$ [ ] by the **fundamental theorem of calculus**.

b) $\int_2^5$ [ ] $dt = f(5) - f(2)$ by the **fundamental theorem of calculus**.

c) A continuous function f for which $f(-1) = -3$ and $f(1) = 8$ has a [ ] by the [ ] theorem.

d) If a differentiable function satisfies $f(a) = f(b)$ then there is a point for $f'(x) = 0$ by the [ ] theorem which is a special case of the mean value theorem.

e) Assume $f_c(x)$ is a **family of functions** such that for $c < 0$, there is no minimum and for $c > 0$ there is one minimum, then c is called a [ ].

Problem 3) Matching (10 points)

Match the following integrals with parts of the regions.

Integral	Fill in 1-6
$\int_1^5 \sin^2(3x)\ dx$	
$\int_1^5 \frac{1}{x^{1/2}}\ dx$	
$\int_1^5 (x-1)e^{-(x-1)^2}\ dx$	
$\int_1^5 \log(x)\ dx$	
$\int_1^5 \frac{x^4-1}{x^4+1}\ dx$	
$\int_1^5 3|\sin(5x)|e^{-x}\ dx$	
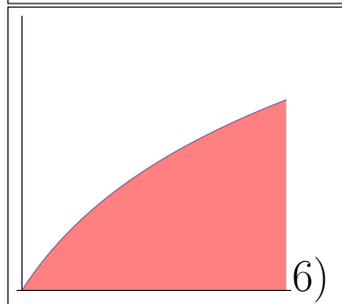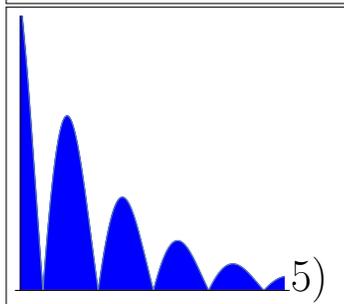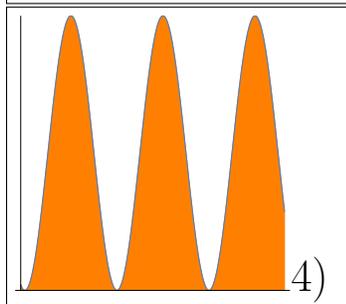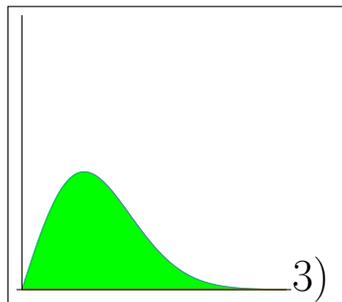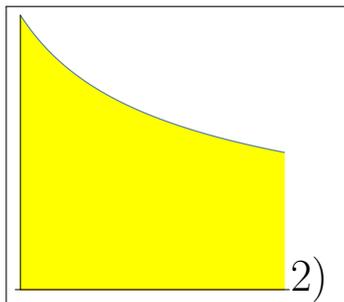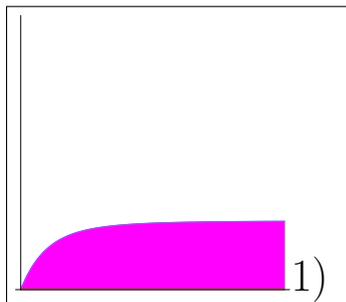
Problem 4) Chain rule (10 points)

Compute the following derivatives:

a) $5\tan(x^2)$

b) $\cos(\sin(3x)) + e^{\cos(x)}$

c) $\cos^7(x) + \cos^3(8x)$

d) $3\sqrt{x^2 + x^5}$

e) $8\ln(2\ln(x^3))$

Problem 5) Related rates (10 points)

A container of length 10, width $2z$ at height z contains water of volume

$$V(z) = 10z^2 .$$

If the volume $V(z(t))$ decreases with constant rate $V' = -1$, how fast does the water level $z(t)$ sink when $t = 1, V = 10$?

You know
$$y^5 x^3 + \cos(x - 2) + y^3 + x = 12$$
and that y is a function of x. Assuming, $x = 2, y = 1$, what is $y'(2)$?

Problem 7) Definite integrals (10 points)

Evaluate the following definite integrals.

a) $\int_0^1 \frac{6}{(x+3)^5} \, dx$

b) $\int_0^1 3x^3/(1+x^4) \, dx$

c) $\int_0^1 17/(1+x^2) \, dx$

d) $\int_0^e \ln(e+x)/(e+x) \, dx$

e) $\int_0^1 99e^{22x} + 5x^7 \, dx$

Problem 8) Anti derivatives (10 points)

Solve the indefinite integrals.

a) $\int 4x^9 + \frac{3}{x} \, dx$

b) $\int \cos(x) + \sin(x) + \tan(x) \, dx$

c) $\int \cos^2(2x) + \sin^2(3x) \, dx$

d) $\int x^6 e^{x^7} \, dx$

e) $\int 3x^2/(1 + x^3) \, dx$

a) Do a Newton step $T(x) = x - f(x)/f'(x)$ to find the root of
$x^7 - x^3 - x = 0$ starting at $x_0 = 1$.

b) Now do a second Newton step.

Problem 10) Catastrophes (10 points)

Consider the family of functions $f(x) = x^3/3 + cx$ on the real line.

a) (4 points) Find all critical points of f for $c < 0$ and determine the stable ones or indicate there are none.

b) (4 points) Find all critical points of f for $c > 0$ and determine the stable ones or indicate there are none.

c) (2 points) For which value of c does a catastrophe occur?