

4/3/2024: Second Hourly

**”By signing, I affirm my awareness of the standards of the
Harvard College Honor Code.”**

Your Name:

Please write neatly. Use the same page for the answer if possible.

1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		100

Problem 1) TF questions (10 points) No justifications are needed.

- 1) T F $\int x^7 dx = 7x^6 + C.$
- 2) T F $\int_{-1}^1 x^7 dx = 0.$
- 3) T F If f is differentiable and $a < b$ are given, then $\int_a^b f'(x) dx / (b - a)$ is the average rate of change $(f(b) - f(a)) / (b - a)$ of f on $[a, b]$.
- 4) T F If $f(x)$ is differentiable, the definite integral $\int_0^1 f(x) dx$ can be approximated by Riemann sums.
- 5) T F If the derivative $g'(x)$ of $g(x) = \int_0^x f(t) dt$ is positive for all x , then $f(x)$ is positive for all x .
- 6) T F If f is differentiable, then $\int_a^b (-f(x)) dx = -\int_a^b f(x) dx.$
- 7) T F The fundamental theorem of calculus implies $\int_a^b f(x) dx = f'(b) - f'(a)$ if f is a differentiable function.
- 8) T F The family $f_c(x) = c(x - 1)^4$ experiences a catastrophe at $c = 0$.
- 9) T F The function $x \ln(x)$ is an anti-derivative of $\ln(x)$.
- 10) T F If f is continuous, then $\int_a^b f(-x) dx = -\int_a^b f(x) dx.$

Problem 2) Theorems (10 points) No justifications are needed.

Fill in the empty box to make a true statement:

a) $\frac{d}{dx} \int_3^x f(t) dt =$ by the fundamental theorem of calculus.

b) Applying an iteration step $T(x) = x - f(x)/f'(x)$ is called a Newton step. It is used to get closer to a of f .

c) A critical point that is a minimum is also called a equilibrium.

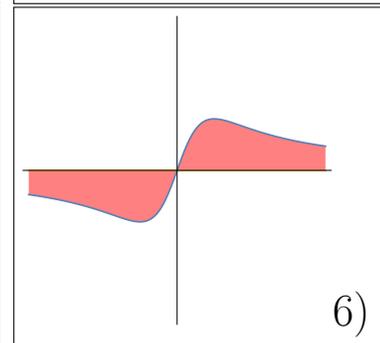
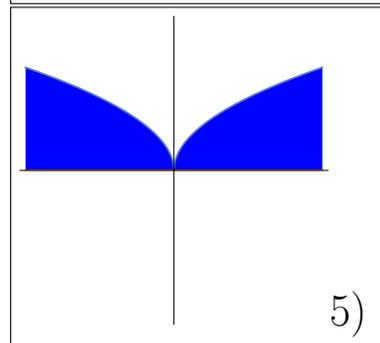
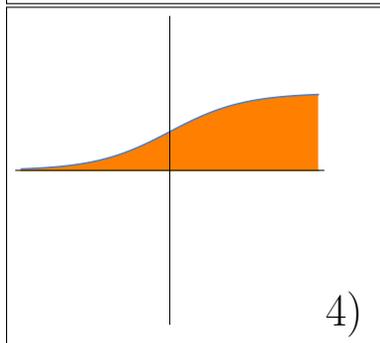
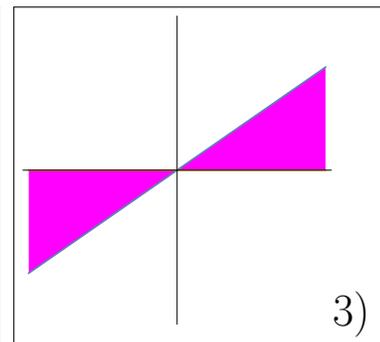
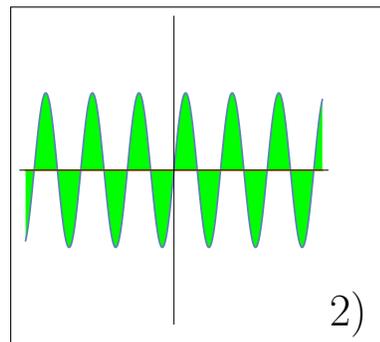
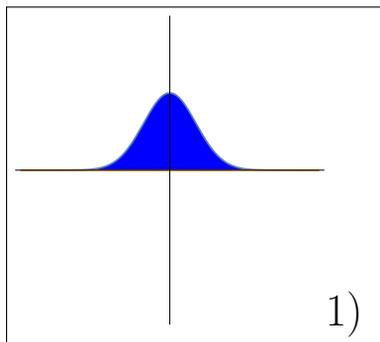
d) If f is differentiable and $f(-3) = 0$ and $f(3) = 1$, then there is a point x in $(-3, 3)$ for which $f'(x) =$.

e) Assume $f_c(x)$ is a **family of functions** such that for $c < 10$, there are exactly 7 minima and for $c > 10$ there are exactly 6 minima, then $c = 10$ is called a .

Problem 3) Matching (10 points)

Match the following integrals with parts of the regions. No justification is needed.

Integral	Fill in 1-6
$\int_{-4}^4 \frac{x}{2} dx$	
$\int_{-4}^4 \frac{e^x}{1+e^x} dx$	
$\int_{-4}^4 \sqrt{ x } dx$	
$\int_{-4}^4 \sin(5x) dx$	
$\int_{-4}^4 e^{-x^2} dx$	
$\int_{-4}^4 \frac{x}{1+x^2} dx$	



Problem 4) Chain rule (10 points)

Compute the derivatives $f'(x)$ of the following functions $f(x)$.
Each problem is 2 points.

a) $f(x) = \sin(x^9)$

b) $f(x) = e^{\sin(x)}$

c) $f(x) = \cos^5(x)$

d) $f(x) = \tan(\sin(x^2))$

e) $f(x) = \ln(\ln(\ln(x)))$

Problem 5) Related rates (10 points)

We know that

$$yx^4 + xy^4 + x^2y^2 = 22$$

relates two functions $x = x(t)$, $y = y(t)$ and that $x' = x'(0) = 2$ and $x = x(0) = 2$, $y = y(0) = 1$.

What is $y' = y'(0)$? ¹

¹As usual, we just always write x, x', y, y' and not $x(0), x'(0), y(0), y'(0)$.

Problem 6) Implicit Differentiation (10 points)

You are given that

$$x^2y^2 - xy + x^8 = 3$$

defines a function $y = y(x)$ near $x = 1, y = 2$. What is $y' = y'(1)$? ²

²As usual, we just always write y, y' and not $y(1), y'(1)$.

Problem 7) Definite integrals (10 points)

Compute the following definite integrals. Each problem is 2 points.

a) $\int_0^1 \frac{1}{x+3} dx$

b) $\int_0^1 \frac{2x}{1+x^2} dx$

c) $\int_0^1 \frac{2}{1+x^2} dx$

d) $\int_0^1 \frac{4x^3}{1+x^4} dx$

e) $\int_0^1 7e^{5x+1} dx$

Problem 8) Indefinite integrals (10 points)

Solve the indefinite integrals. Each problem is 2 points.

a) $\int 3x^2 + 4x^3 dx$

b) $\int 2 \cos(x) + 3 \sin(x) dx$

c) $\int \tan(2x) dx$

d) $\int \sin(8x^2)x dx$

e) $\int \frac{e^x}{1+e^x} dx$

Problem 9) Newton Step (10 points)

a) (5 points) Do a Newton step

$$T(x) = x - \frac{f(x)}{f'(x)}$$

with the function $f(x) = x^3 - 7 = 0$ starting at $x_0 = 2$.

b) (5 points) Now do a second Newton step.

Problem 10) Catastrophes (10 points)

Let $f_c(x) = cx^2 - x^2 + x^3$.

- a) (3 points) Verify that $x = 0$ is a critical point for all c .
- b) (4 points) Determine the stability of this critical point $x = 0$ depending on c .
- c) (3 points) For which value of c does a catastrophe appear?