# Math1a Spring 2011

# Final Review

May 9, 2011

# Functions

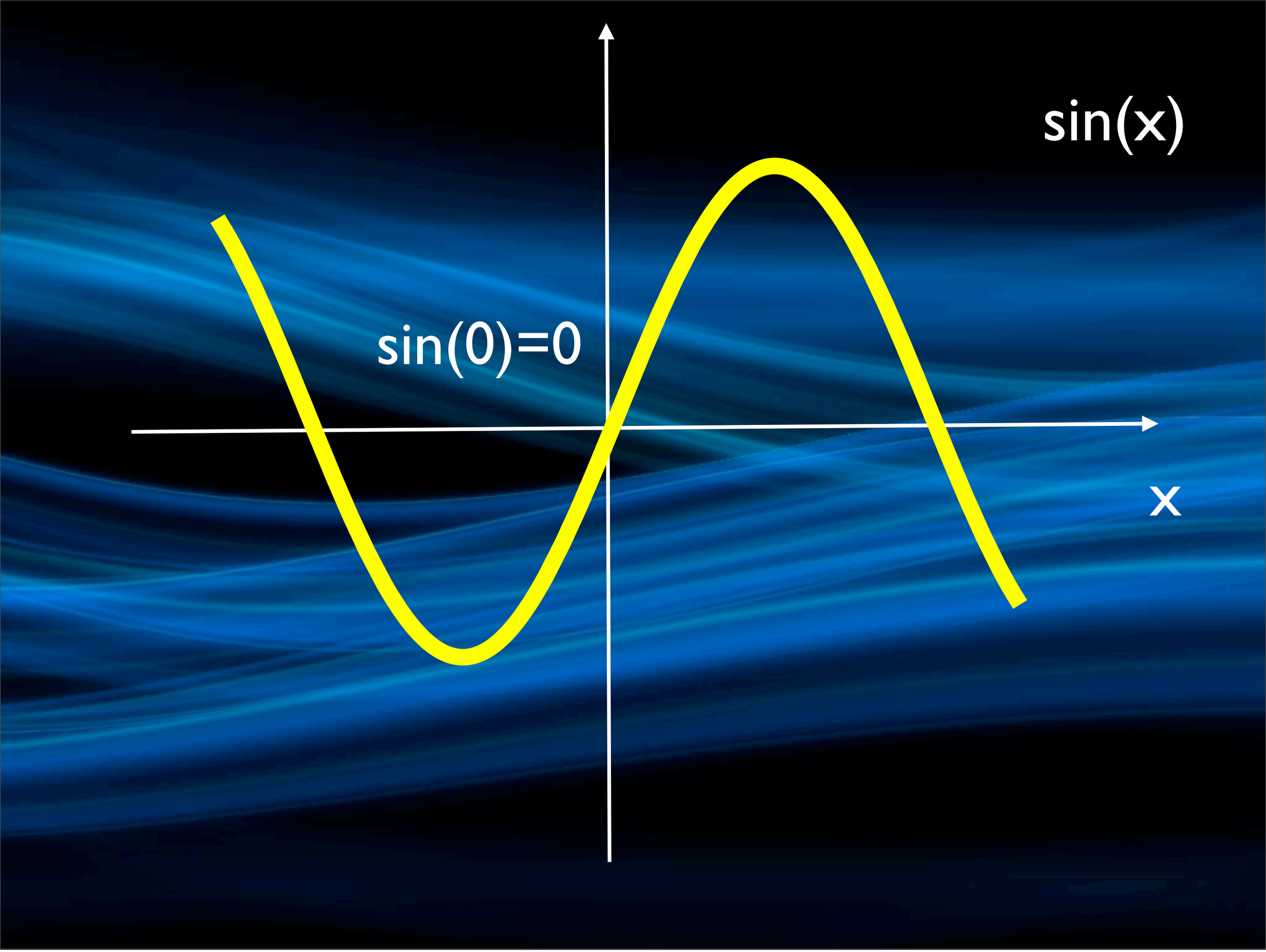
# Important Functions

- sin
- cos
- tan
- arcsin
- arccos
- arctan

- polynomials
- rational functions
- trig polynomial

- exp
- log
- sqt = √

Add
Subtract
Compose
Multiply
Divide


sin(x)
sin(0)=0
x

cos(x)

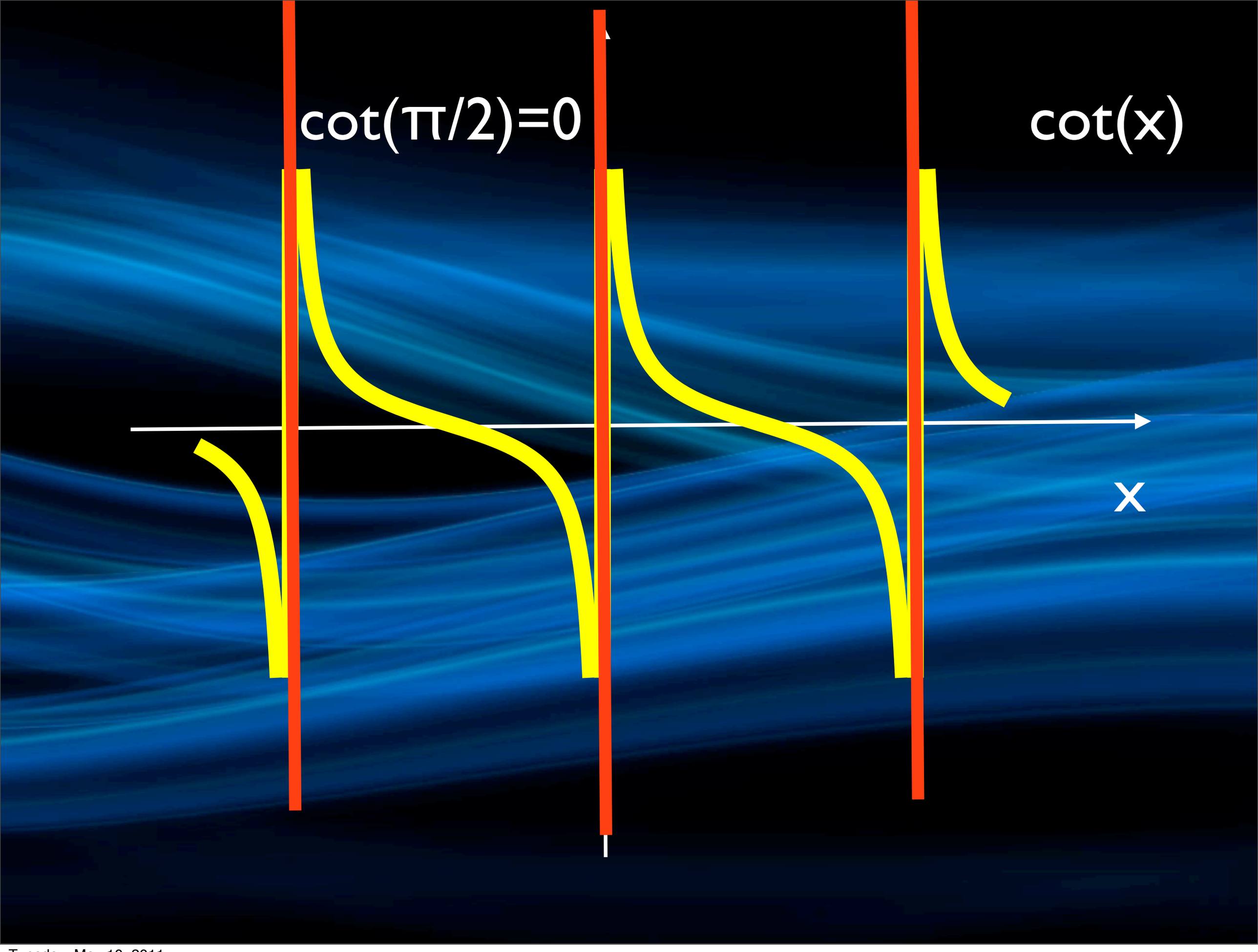
cos(0)=1

x

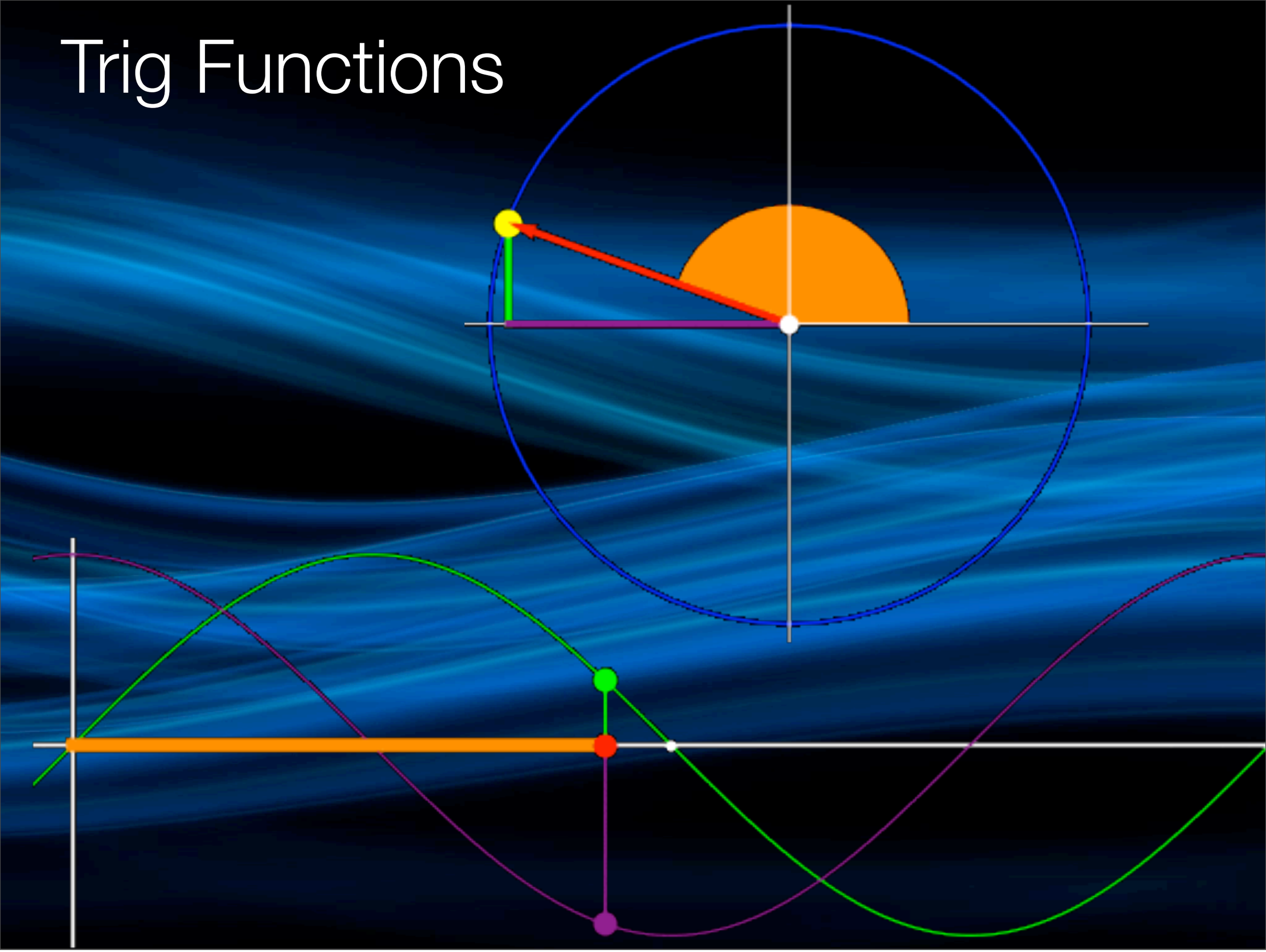
tan(x)

tan(0)=0

x


cot(π/2)=0
cot(x)
x

# Trig Functions



# Problem 1

$\sin(5\ \pi/4) = ?$

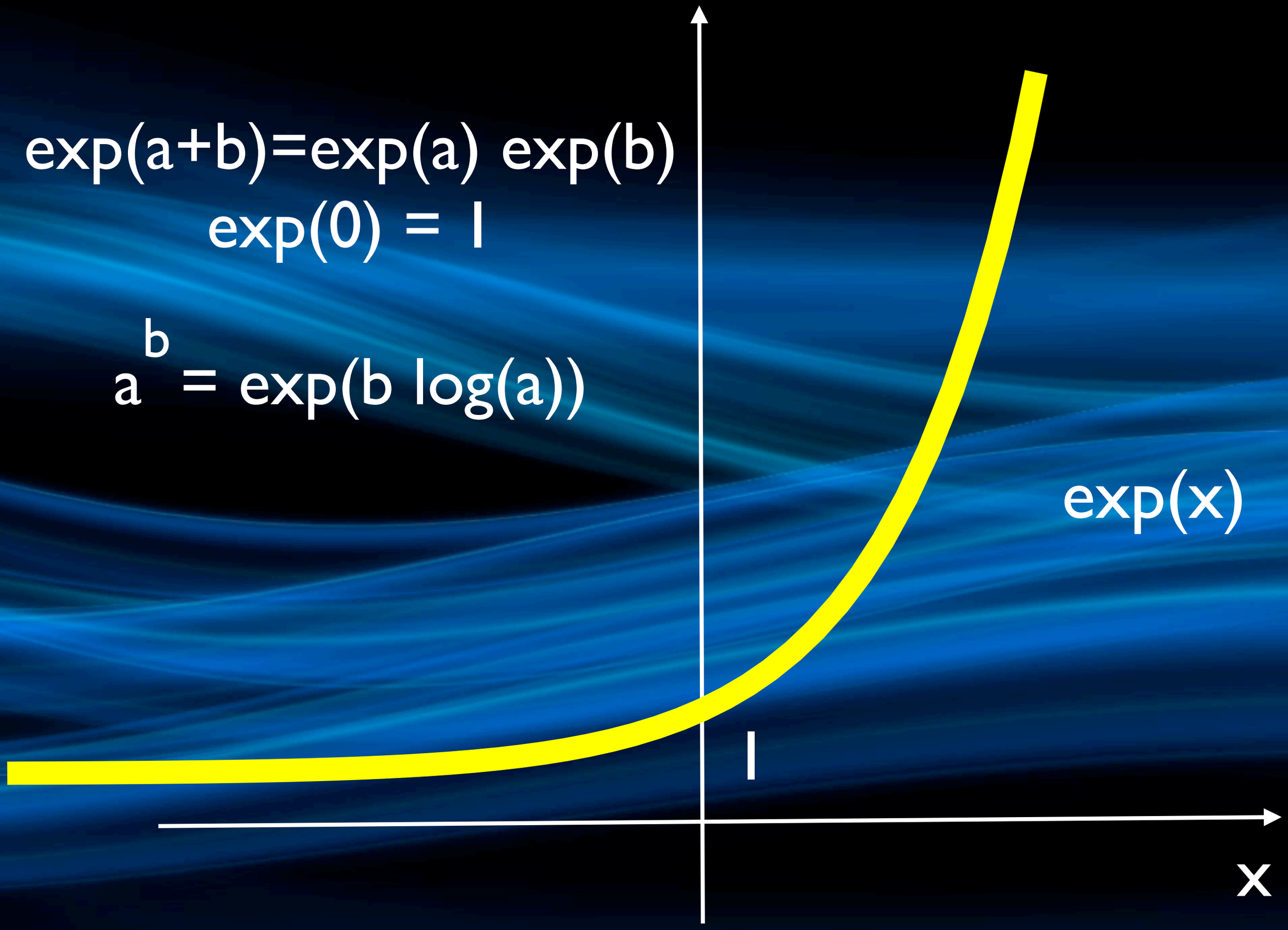
$\cos(3\ \pi/2) = ?$

$\tan(-\pi/6) = ?$

$$\exp(a+b)=\exp(a)\,\exp(b)$$

$$\exp(0) = 1$$

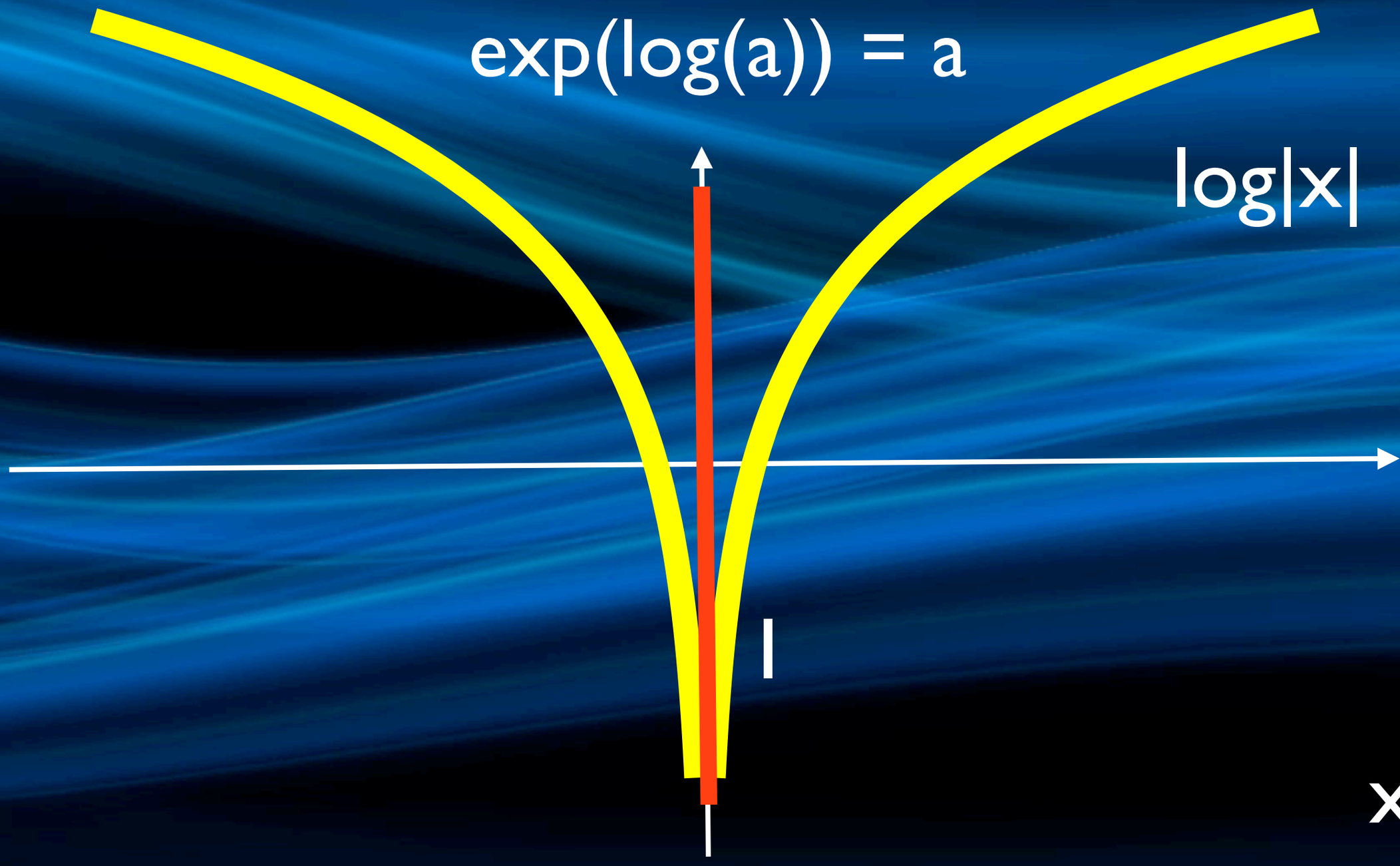
$$a^{b} = \exp(b \log(a))$$



$$\log(a\ b) = \log(a) + \log(b)$$

$$\log(1) = 0$$

$$\exp(\log(a)) = a$$

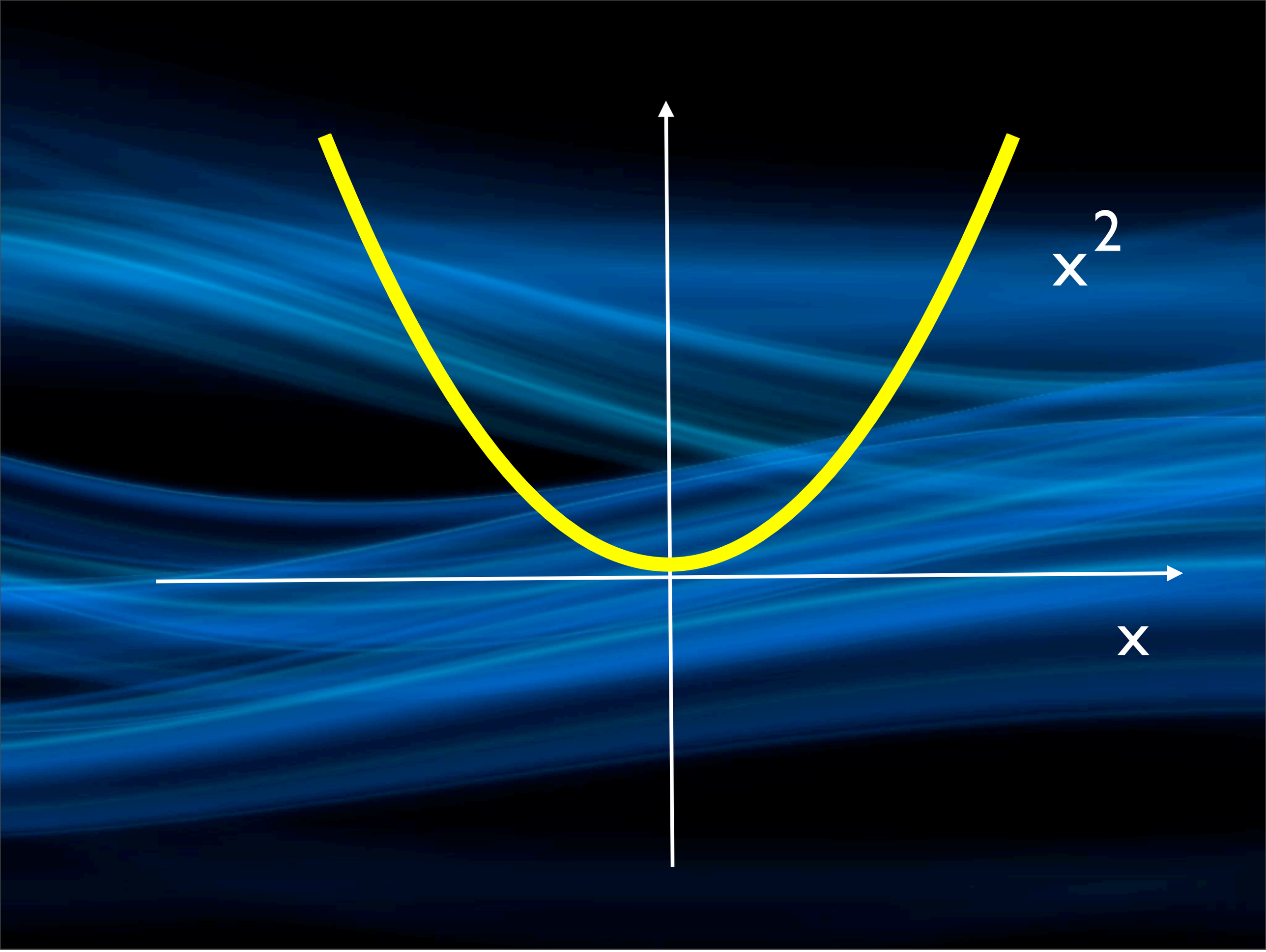


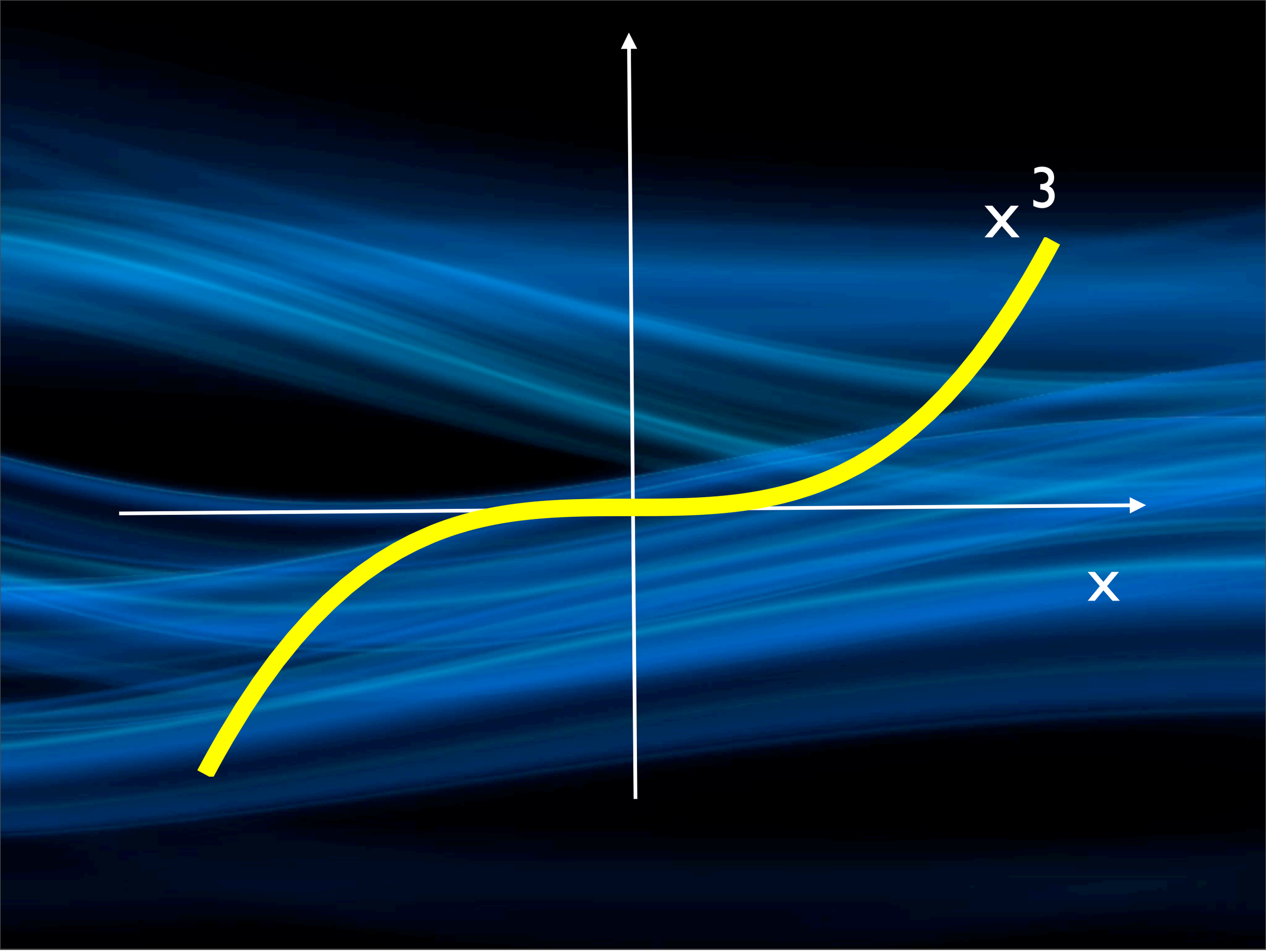
# Problem 2

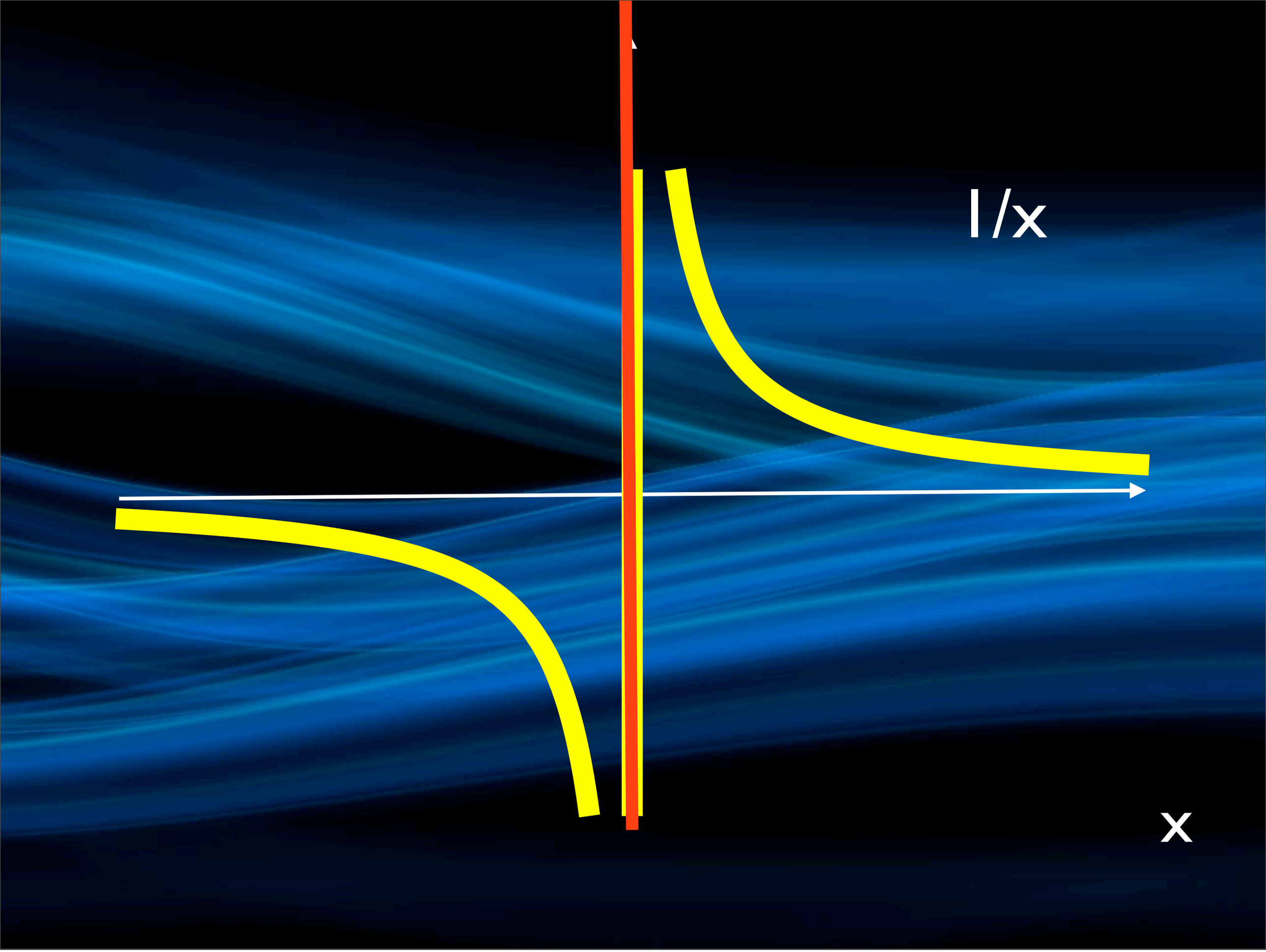
$$\exp(0)+\exp(1)$$

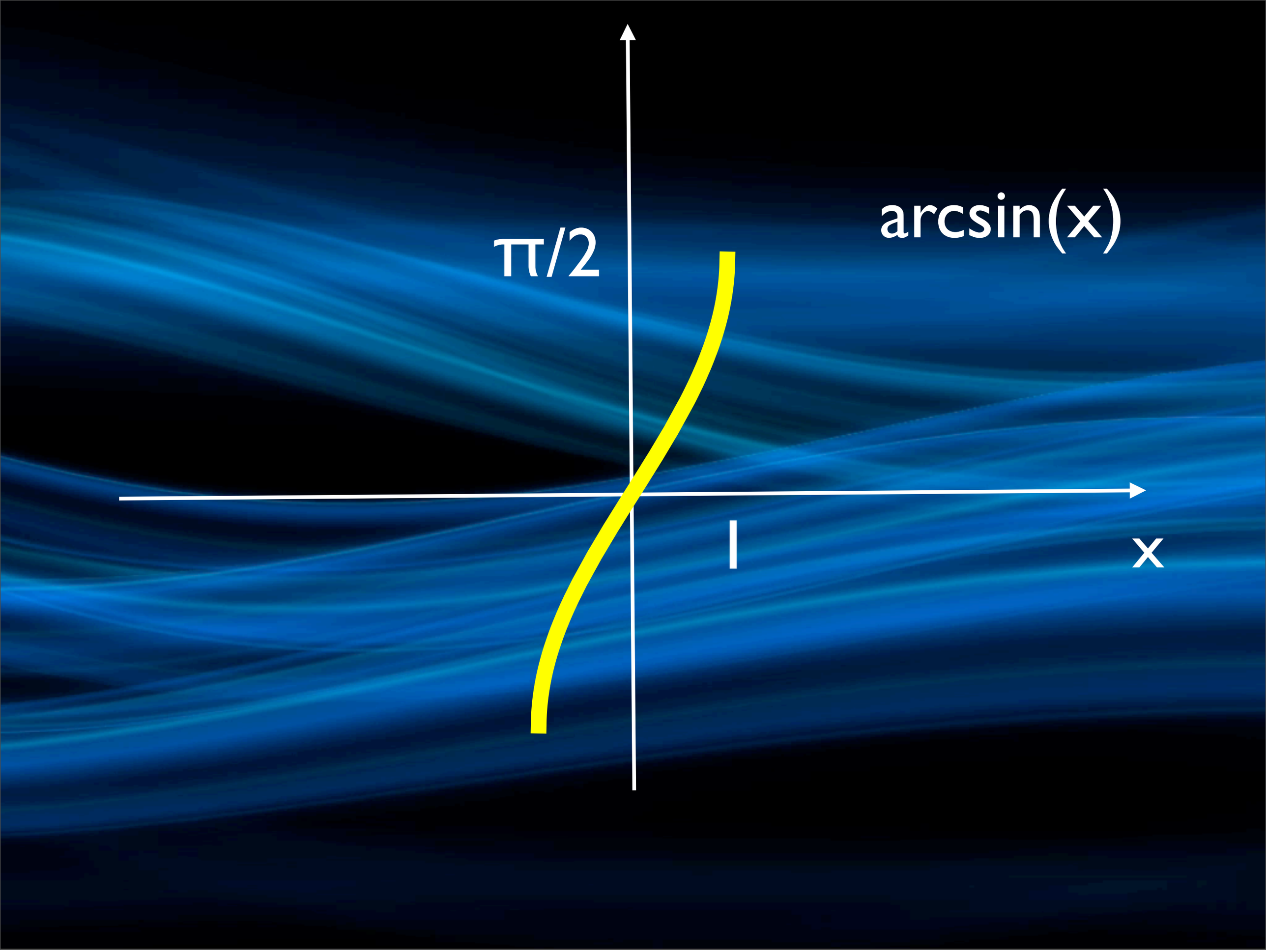
$$\log(e) + \log(1)$$

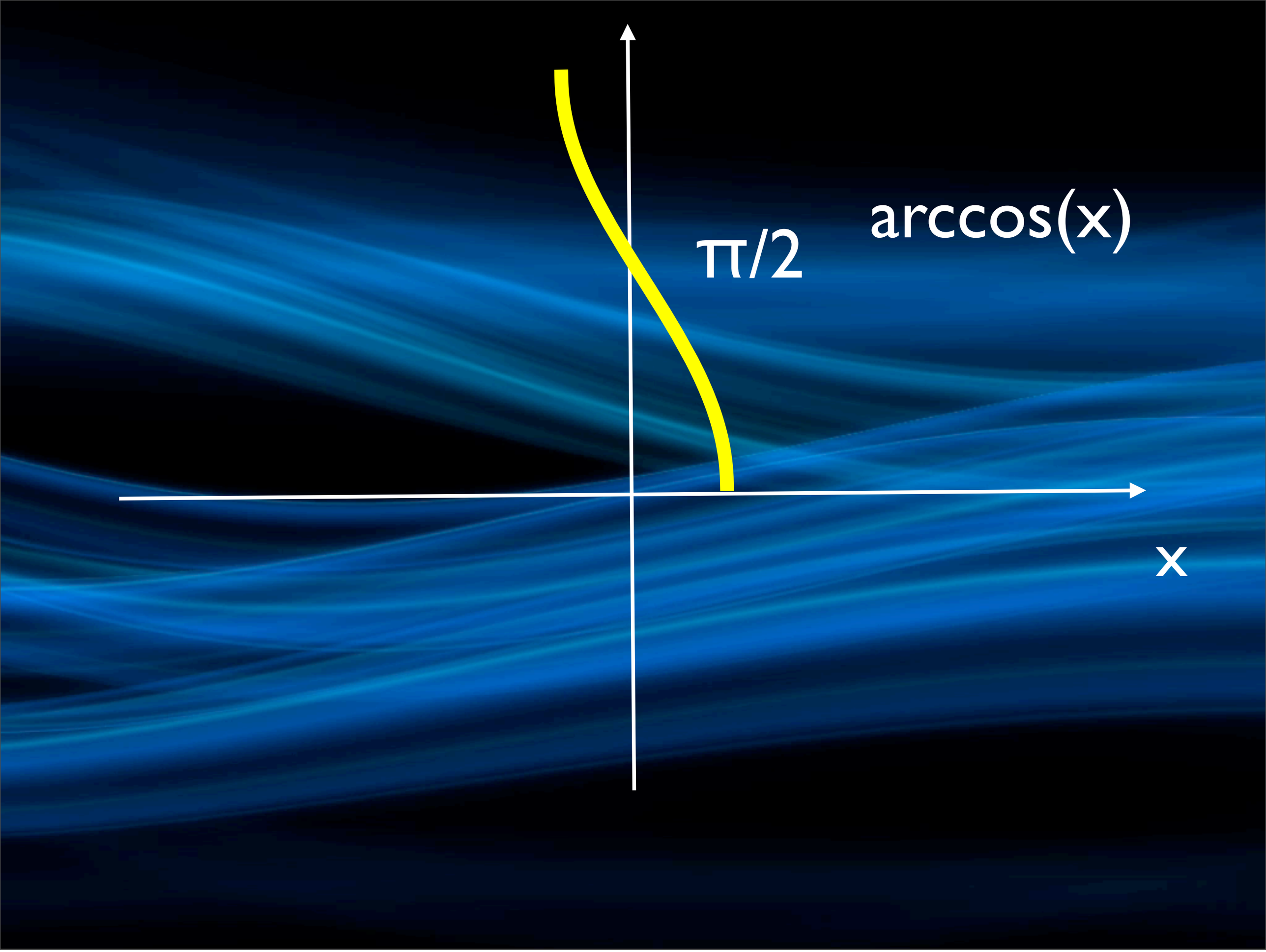
$$e^{\log(2)\ 3}$$

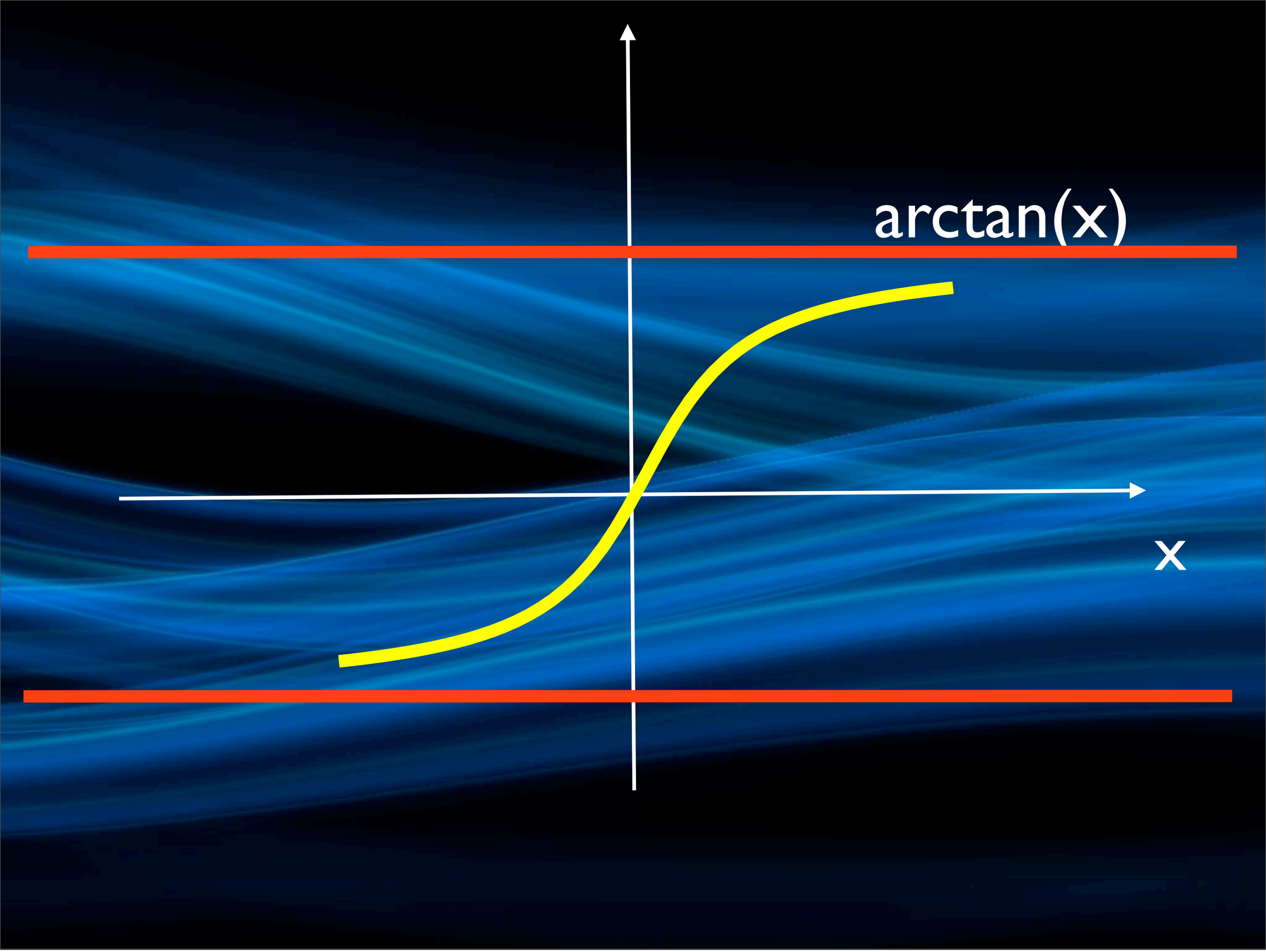

$x^2$
x


x 3
x


1/x
x


π/2
arcsin(x)
1
x


π/2
arccos(x)
x


arctan(x)
x

# Problem 3

$\arcsin(1)=?$

$\arccos(1)=?$

$\arctan(1) = ?$

# Things to look out for:

- even  odd  or neither
- monotoincally increasing, decreasing
- concave up, concave down
- inflection points, roots, critical points

# Limits

# Medicine

Healing
Simplify
Hopital rules



- Hopitals rule 0/0
- Hopitals rule $^{\infty}/_{\infty}$

# Problem 4: find the following limits for x->0

- sin(3x)/x
- $2/x-2x/x^2$
- (exp(4x)-1)/x
- tan(x)/x
- (cos(5x)-1)/x^2

# Problem 5

What is the limit of

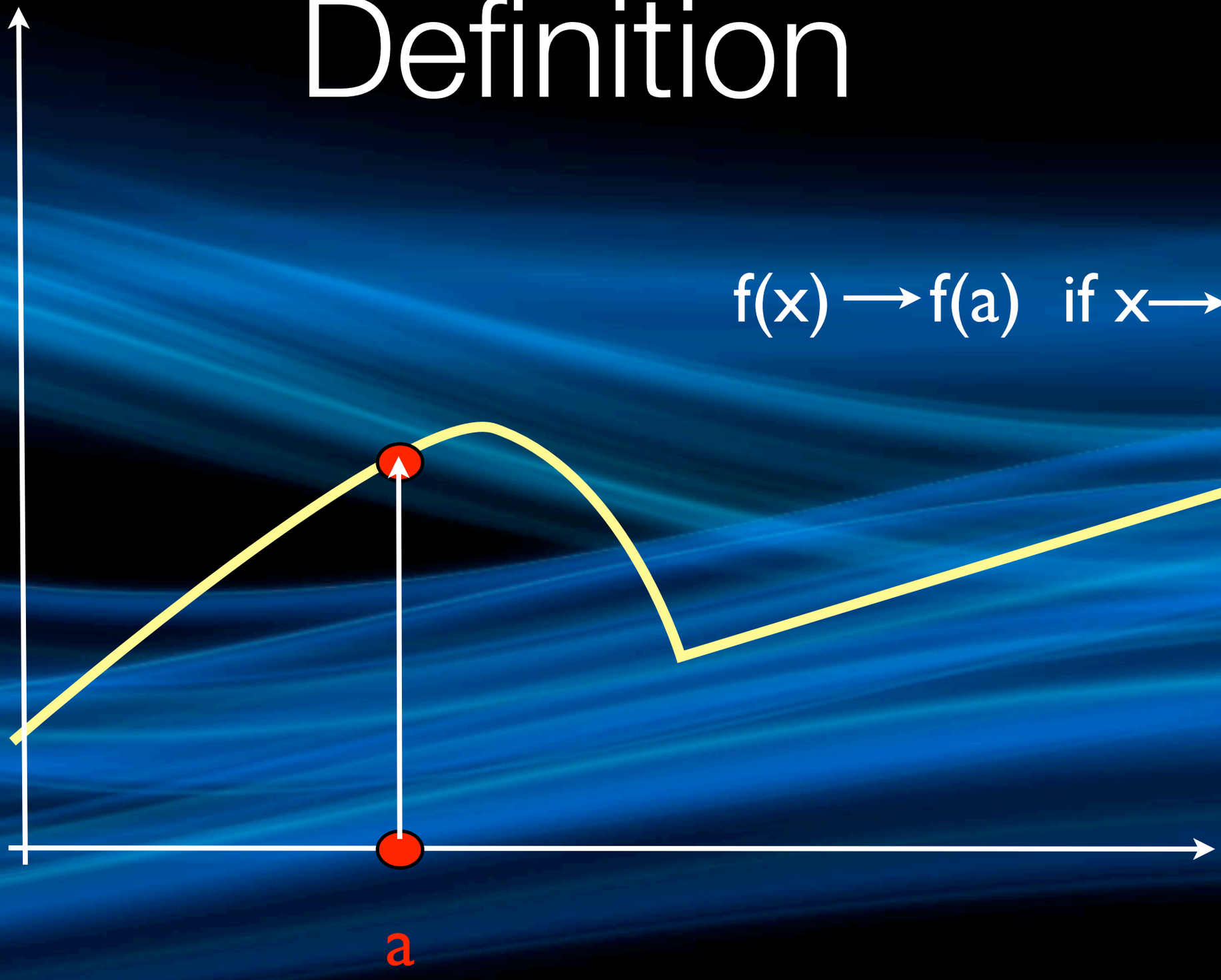
$$\frac{x^{13} - 1}{x - 1}$$

as x goes to 1?

# Continuity

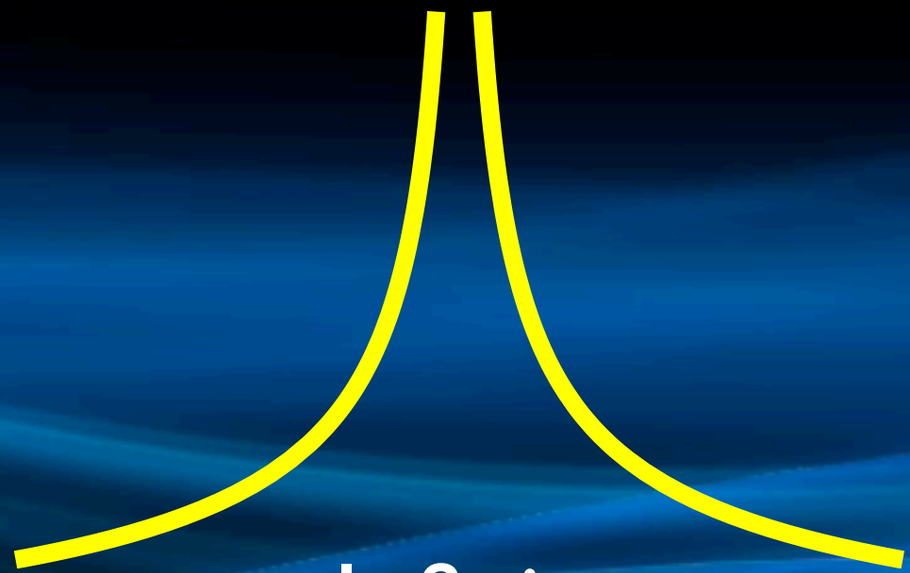
# Definition

$f(x) \to f(a)$ if $x \to a$

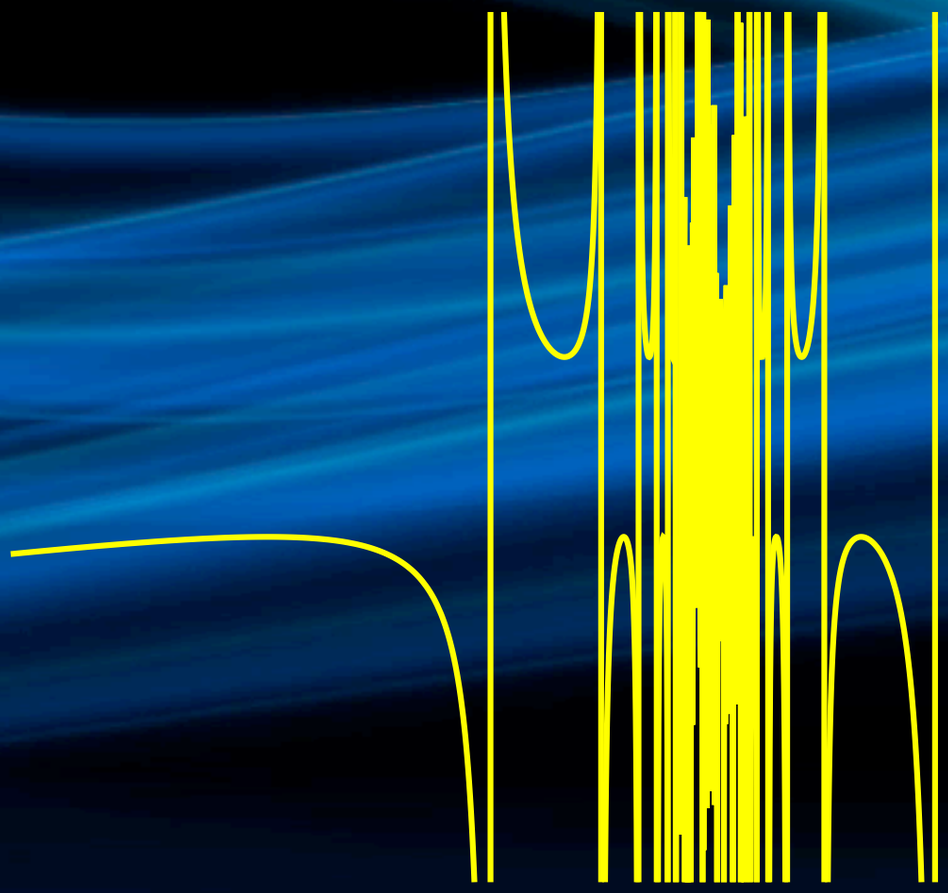


# Discontinuity types

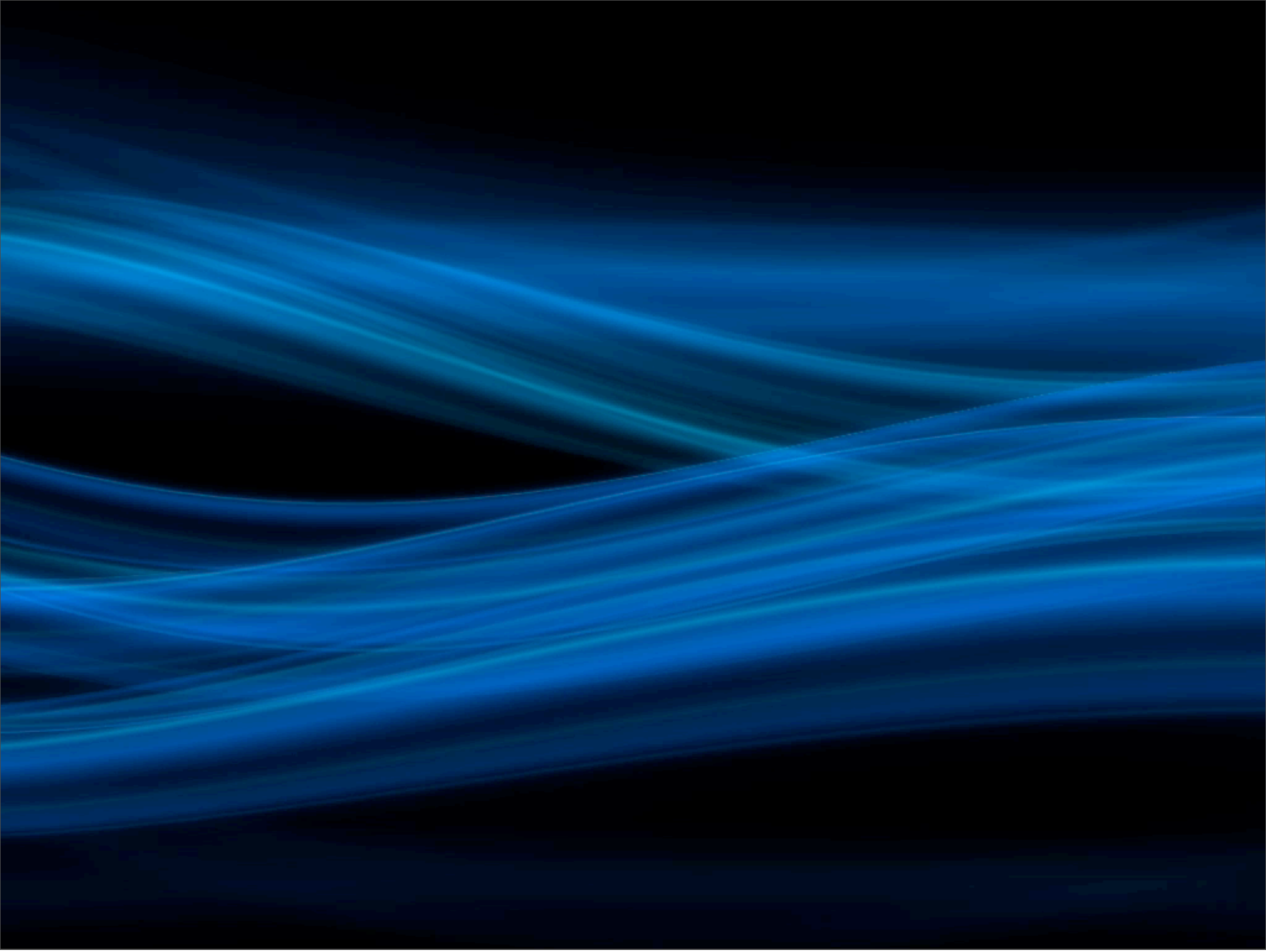
Jump



Infinity



Oscillate



Most functions are nice and continuous

Most functions are
nice and continuous

Be aware of the
mean ones however

# Problem 6

are the following functions continuous?

$$\cos(x)/\log|x|$$

$$\sin(\sin(x))/x$$

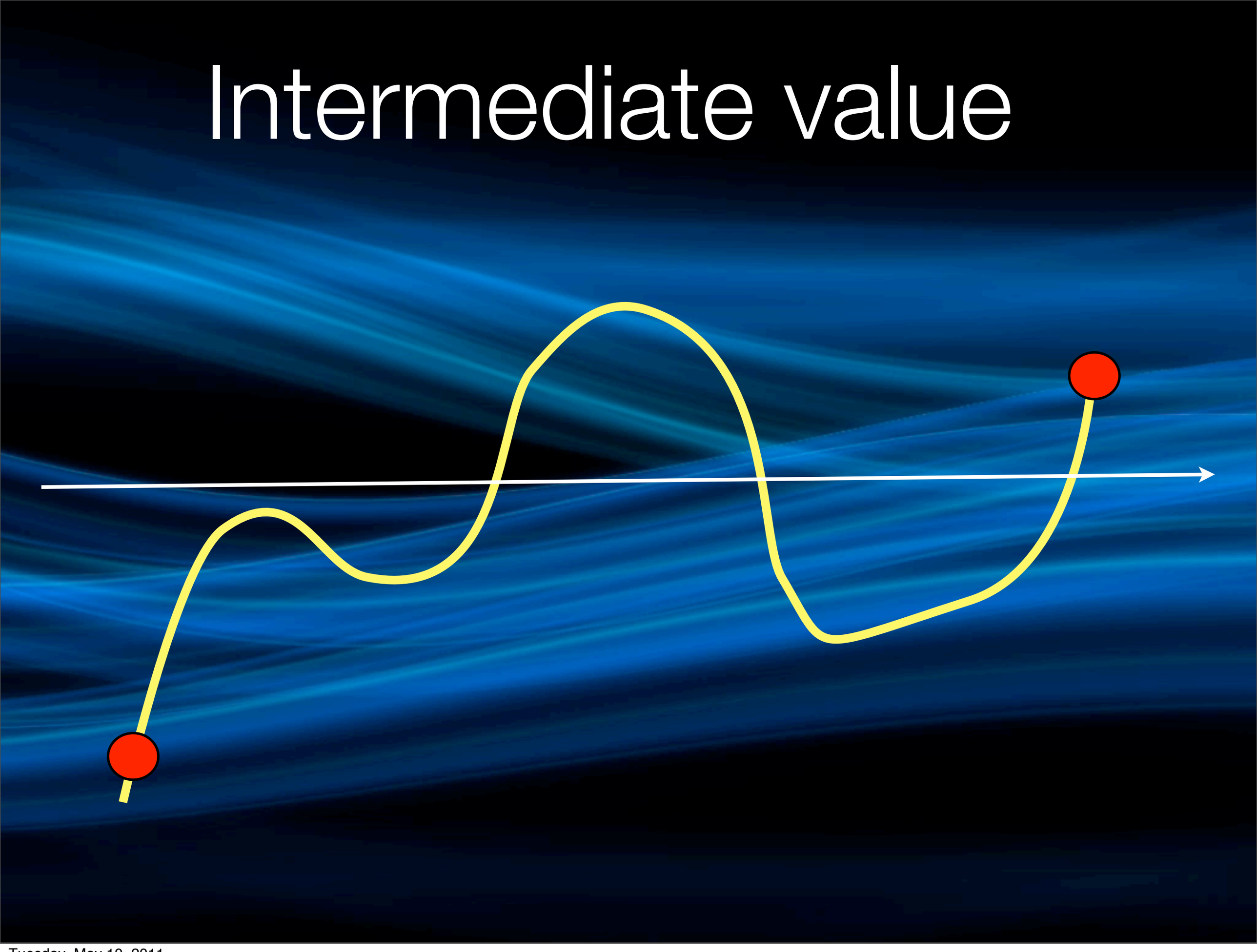
on the real line?

# Theorems

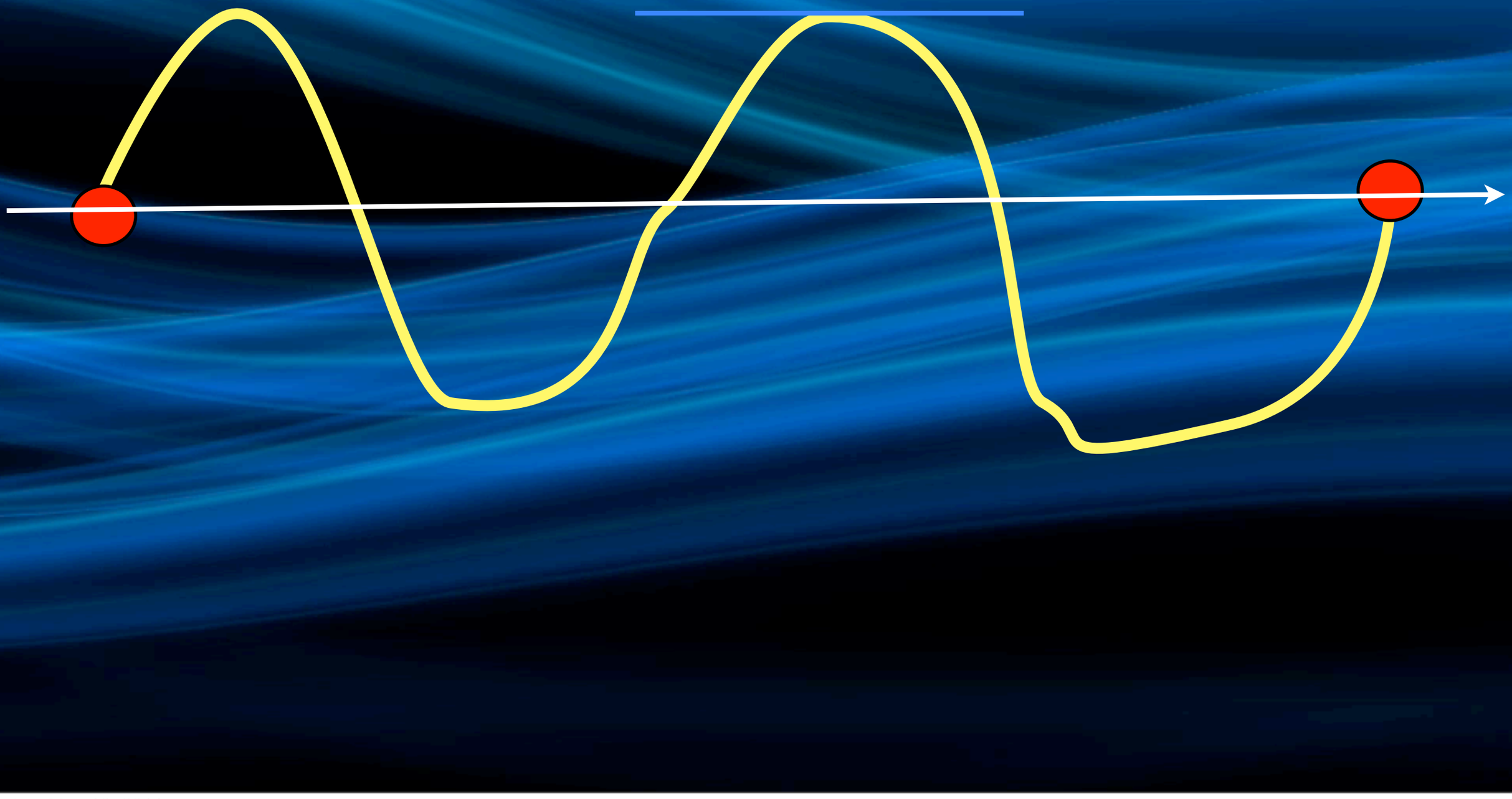
# Theorems on Continuity

- Intermediate Value theorem
- Rolle theorem
- Mean value theorem
- Fermat theorem
- Wobbly Table theorem

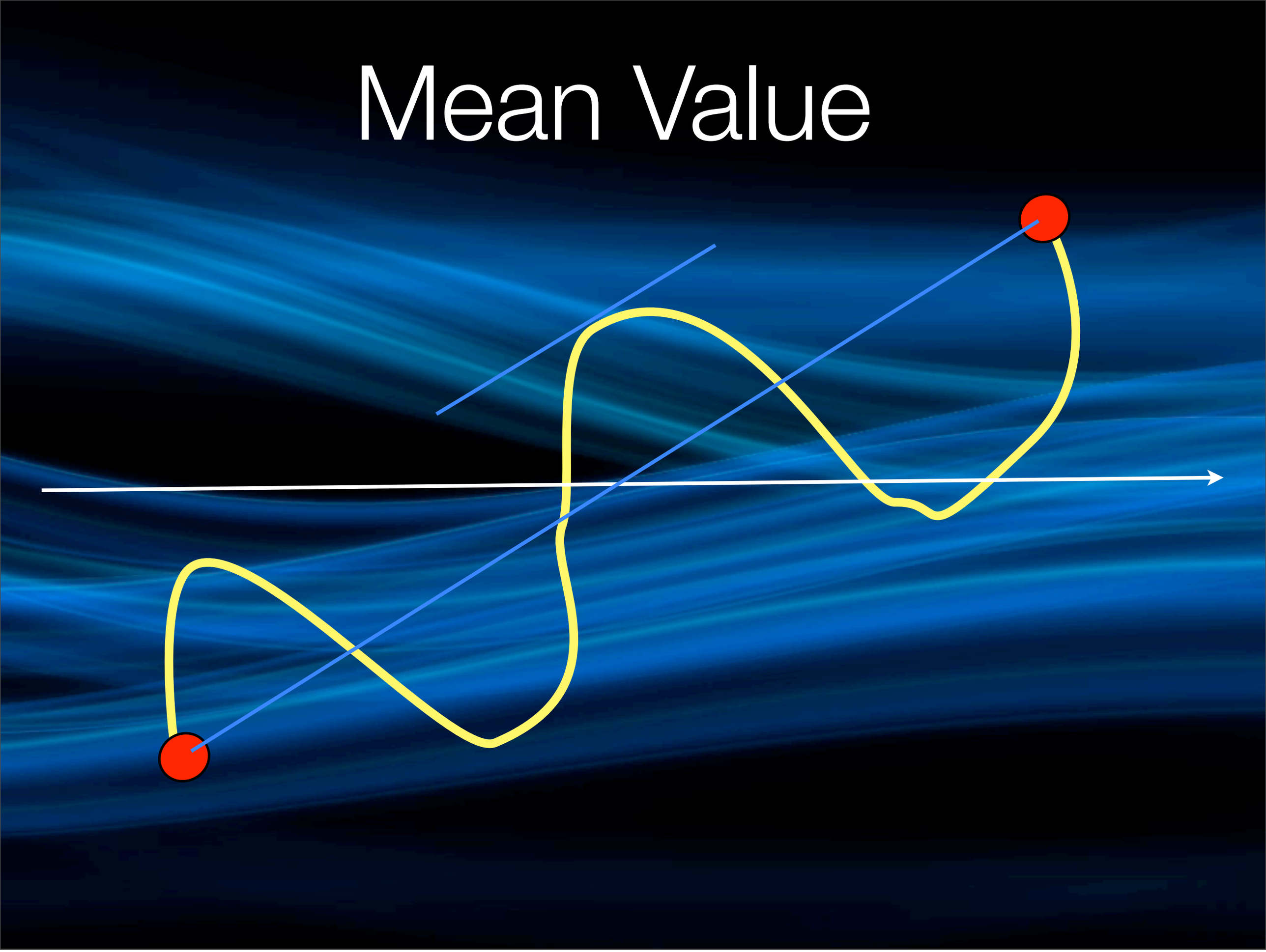
# Intermediate value



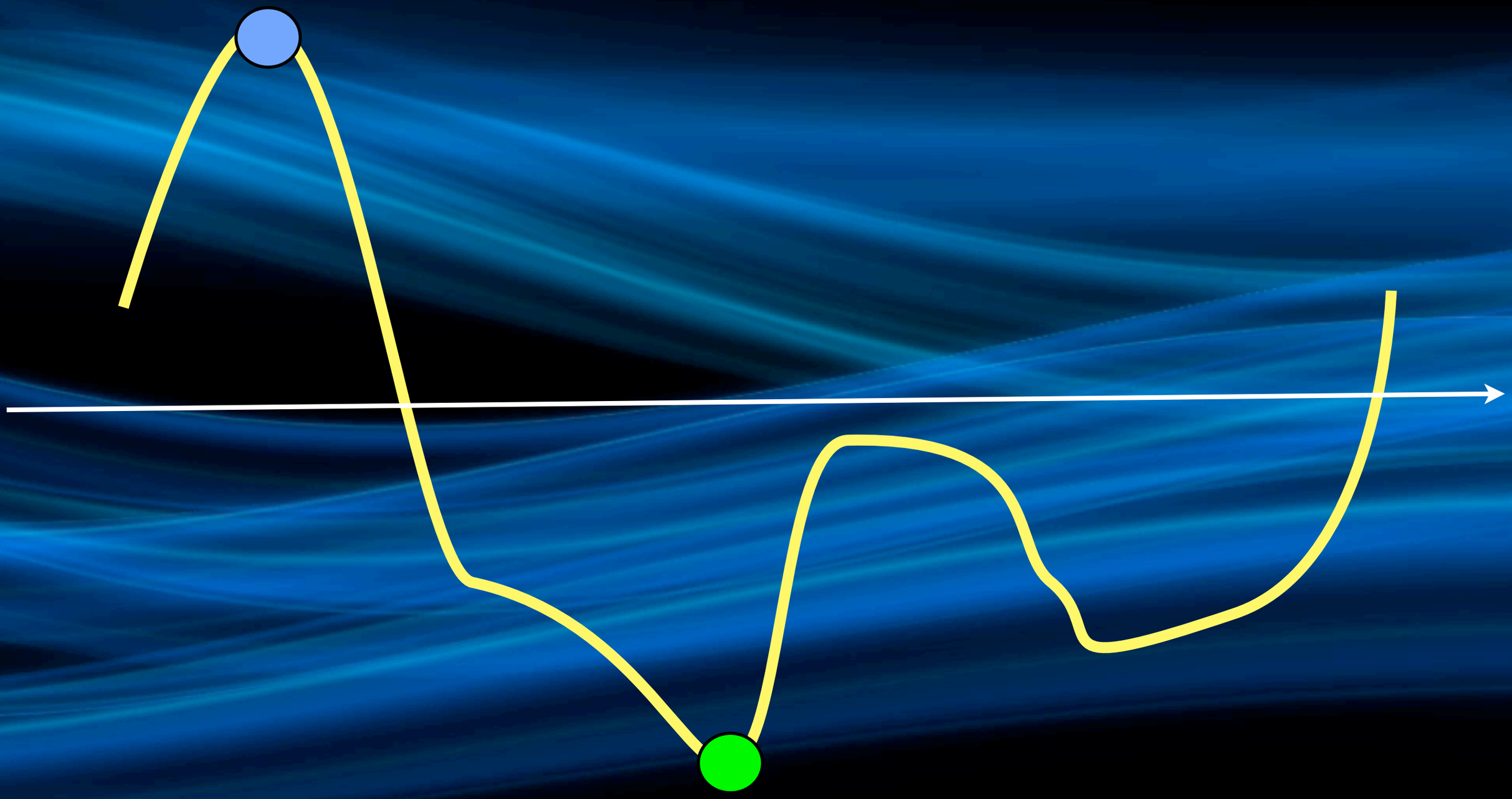
# Rolle



# Mean Value



# Fermat



# The Wobbly Chair Theorem

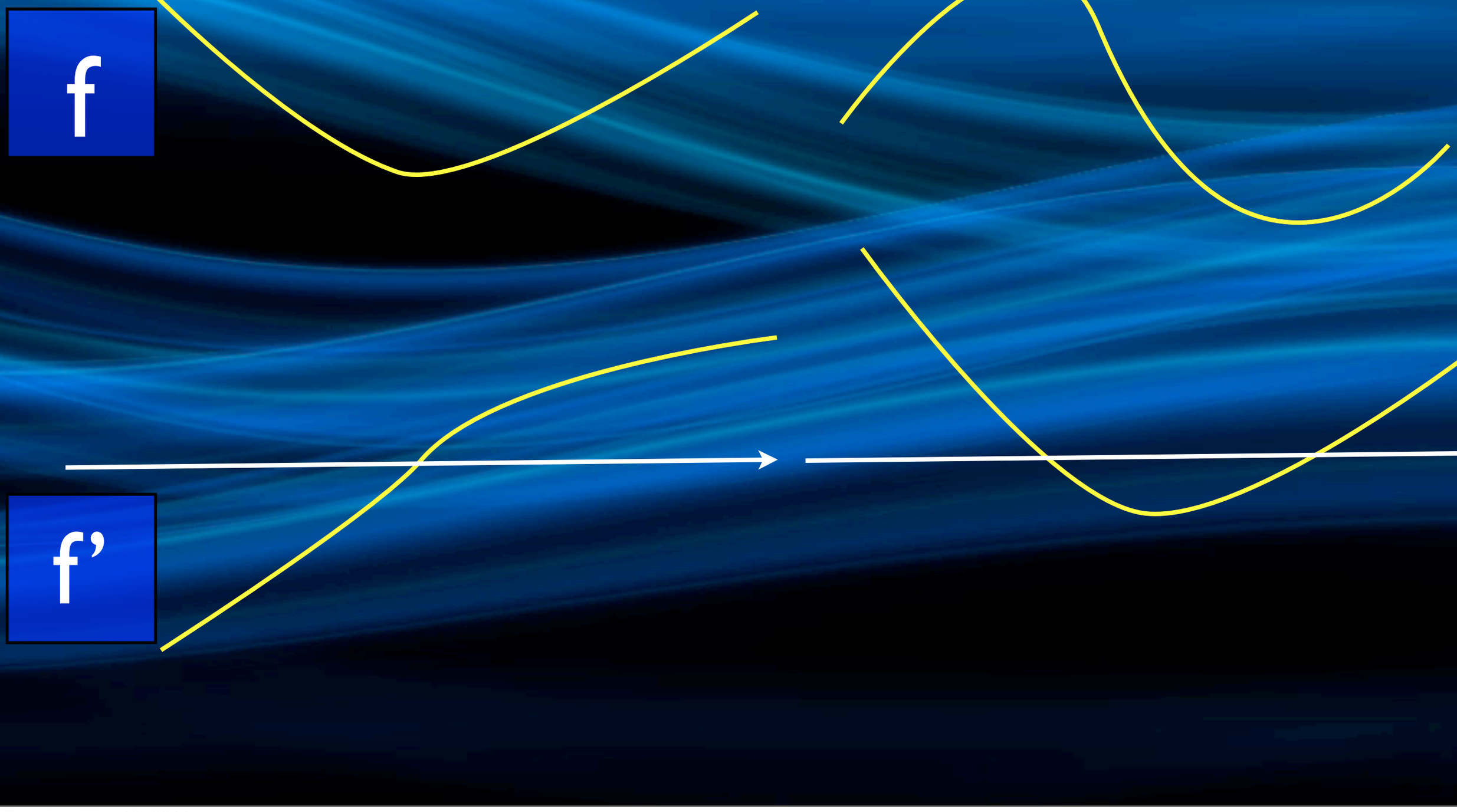
Documenting moment of Discovery:
Sunday, May 8, 2011

# Differentiation

# Differential quotient

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

# Matching Problems

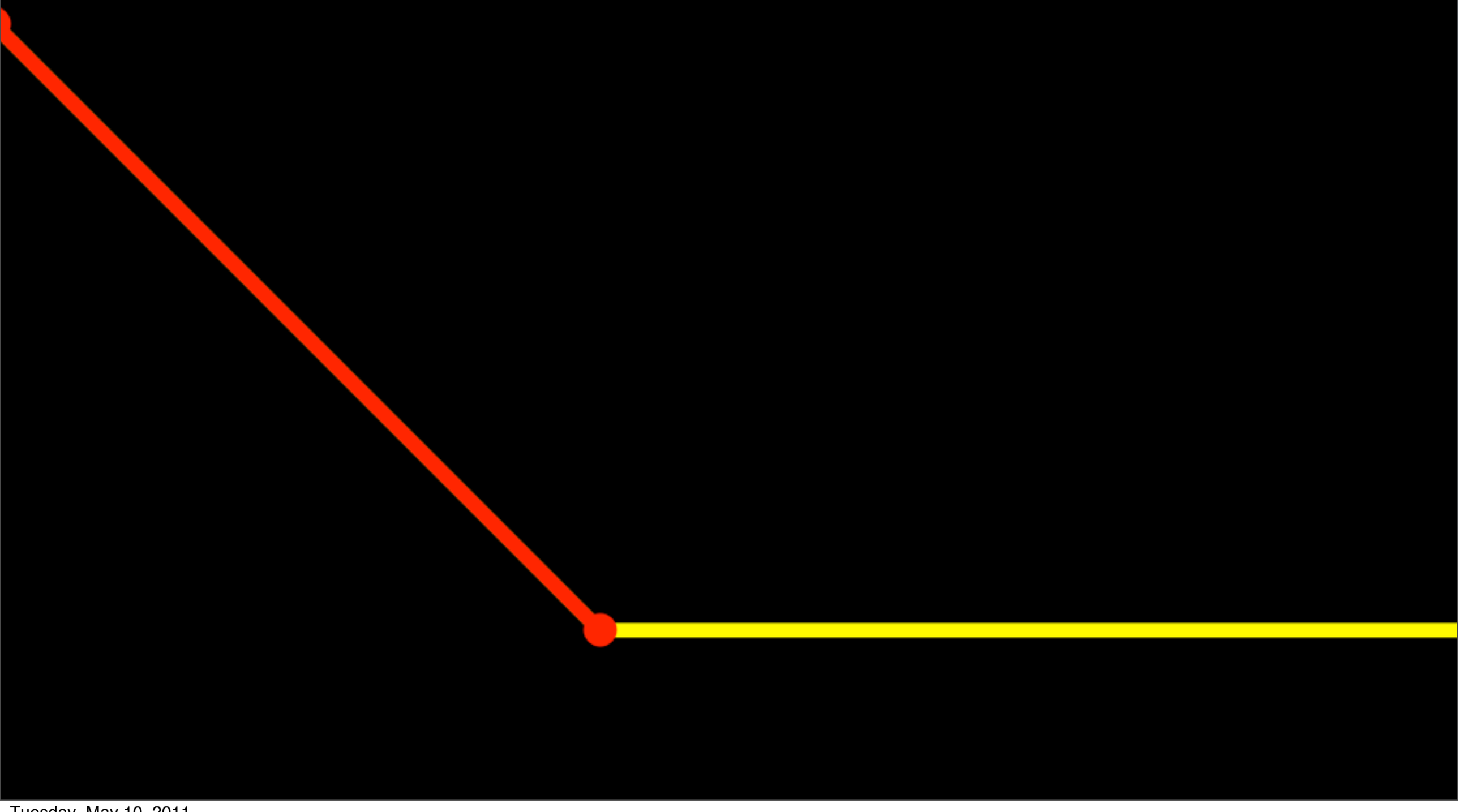


# Matching Problems

- Product rule $(f\ g)' = f'g+fg'$
- Addition rule $(f+g)' = f'+g'$
- Chain rule $f(g)' = f'(g(x))\ g'(x)$
- Quotient rule $(f/g)' = (f'g-fg')/g^2$
- Reciprocal rule $(1/f)' = -f'/f^2$

# Related Rates

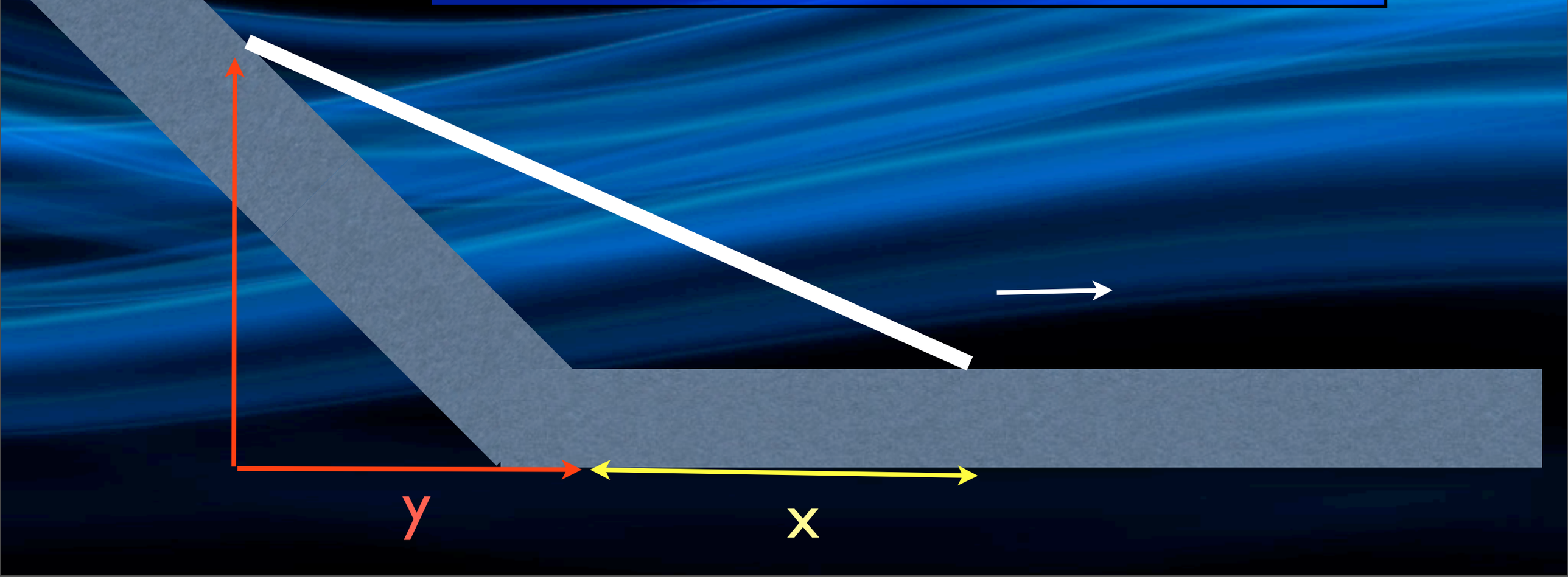
# Problem 7: an other ladder problem



Find y' if x=2,y=1 and x(t)=1+t

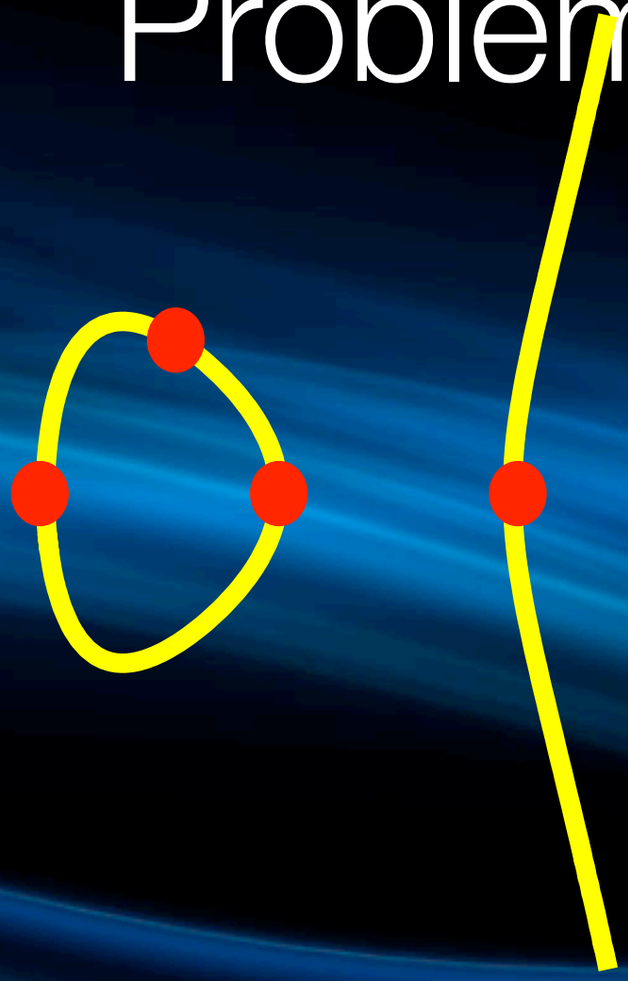
Find y'(t) at t=1.

$$(x+y)^2+y^2=10$$



# Problem 8: Implicit differentiation

## Problem:



Find the slope at x=1/2

Bolza curve

$y^2 = x^5 - x$


Oscar Bolza
1857-1942

# Extremization

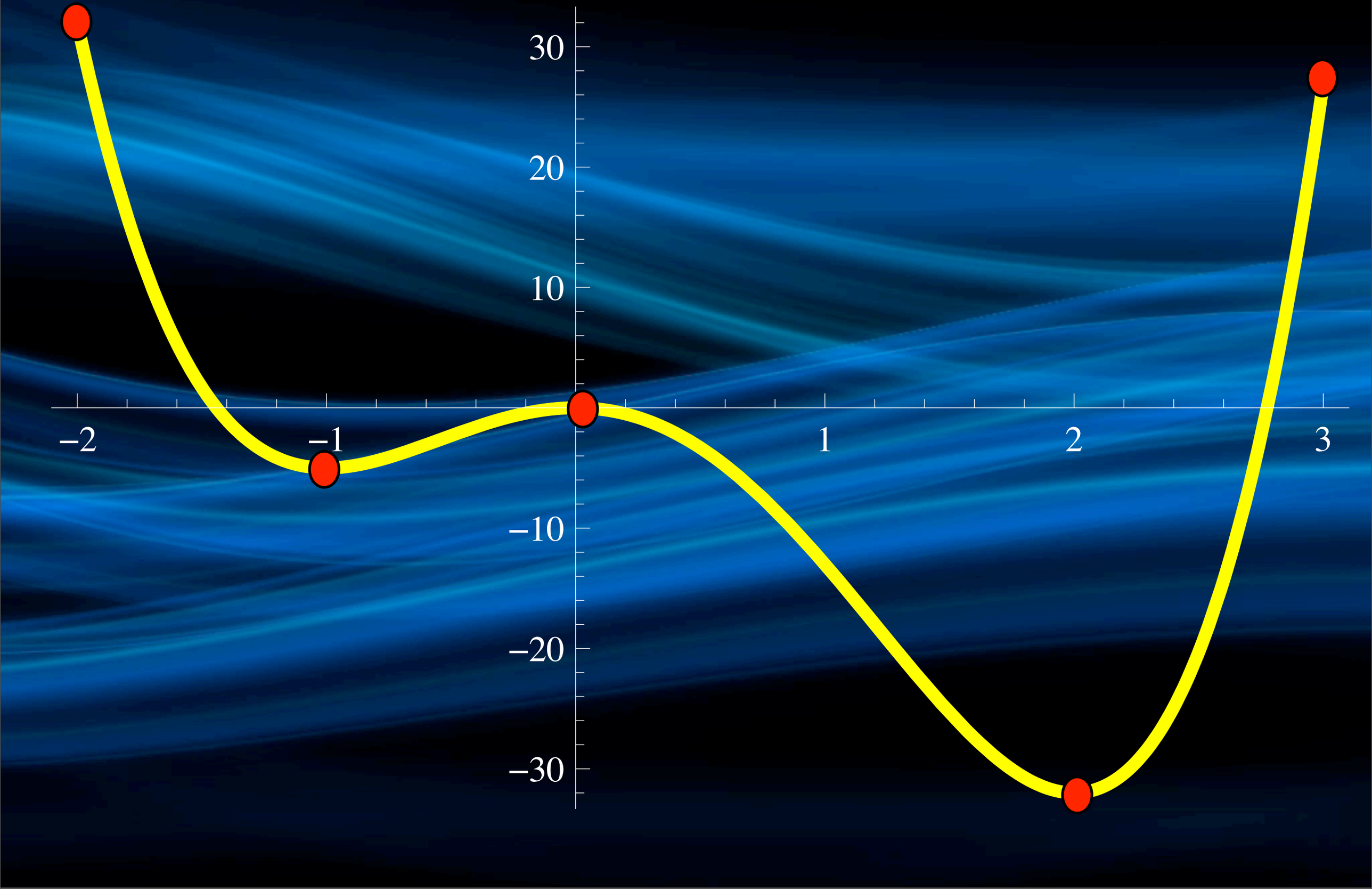
# Problem 9:

Find the local and global maxima

$$-12x - 4x^2 + 3x^3 + 4$$

on the interval [-2,3]

# The function



# Problem 10:


2x+y=10 Find maximal area.

# Integration

# Integration Techniques

- Substitution $\int f(g(x))\, g'(x)\, dx = \int f(u)\, du$
- Parts $\int uv'dx = uv - \int u' v\, dx$
- Partial Fraction $\frac{1}{(x-a)(x-b)} = \frac{1}{(x-a)} + \frac{1}{(x-b)}$
- Trig Substitution $\int \sqrt{1-x^2}\, dx = \int \cos^2(u)\, du$

# Problem 11: Integrate

$$\int (x+1)^3 \exp(x+2)\,dx$$

$$\int \sqrt{1-x^2}\,dx$$

$$\int 1/\cos(x)\,dx$$

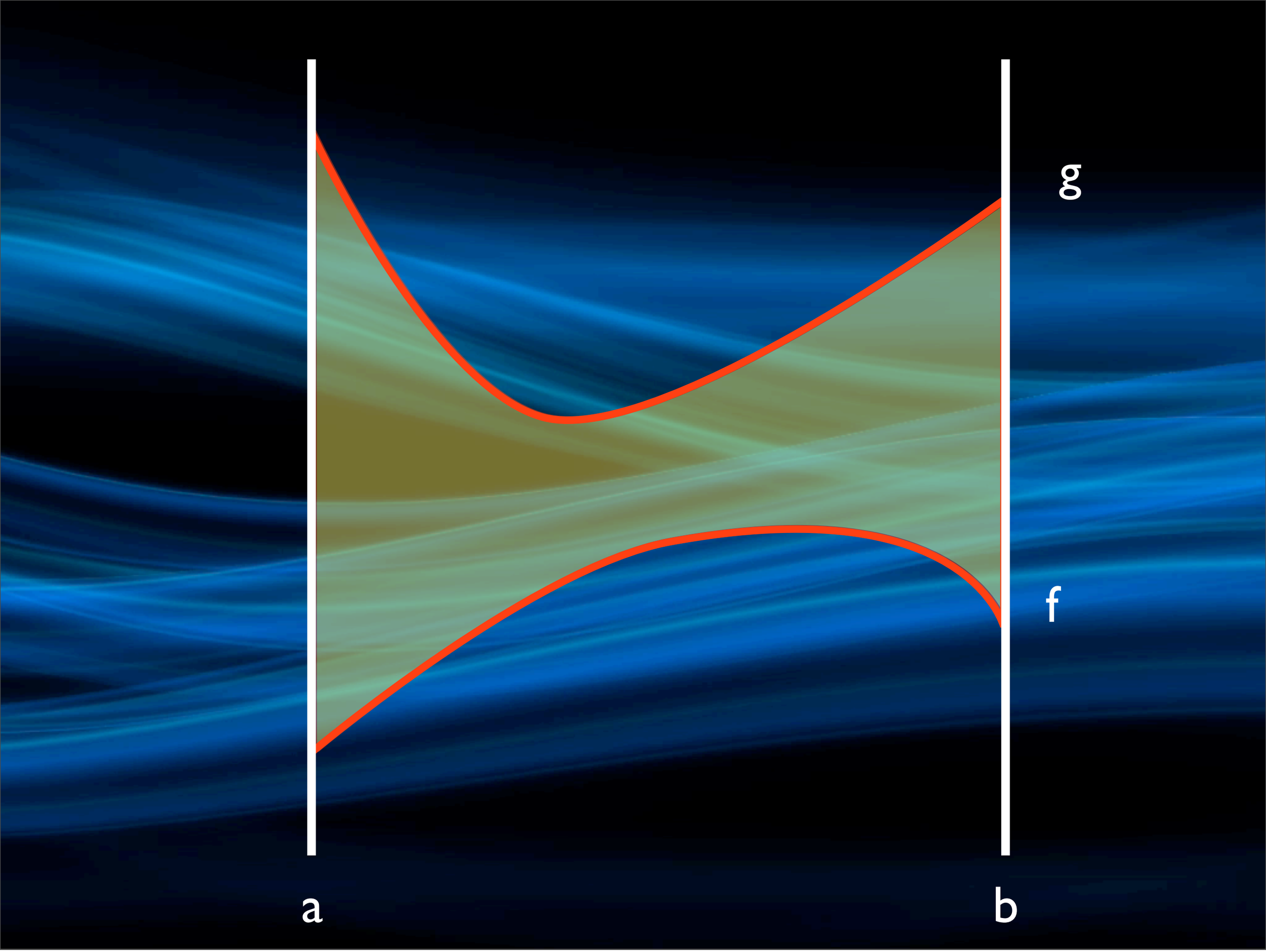
$u=\tan(x/2)$

$dx=2du/(1+u^2)$

$\sin(x)=2u/(1+u^2)$

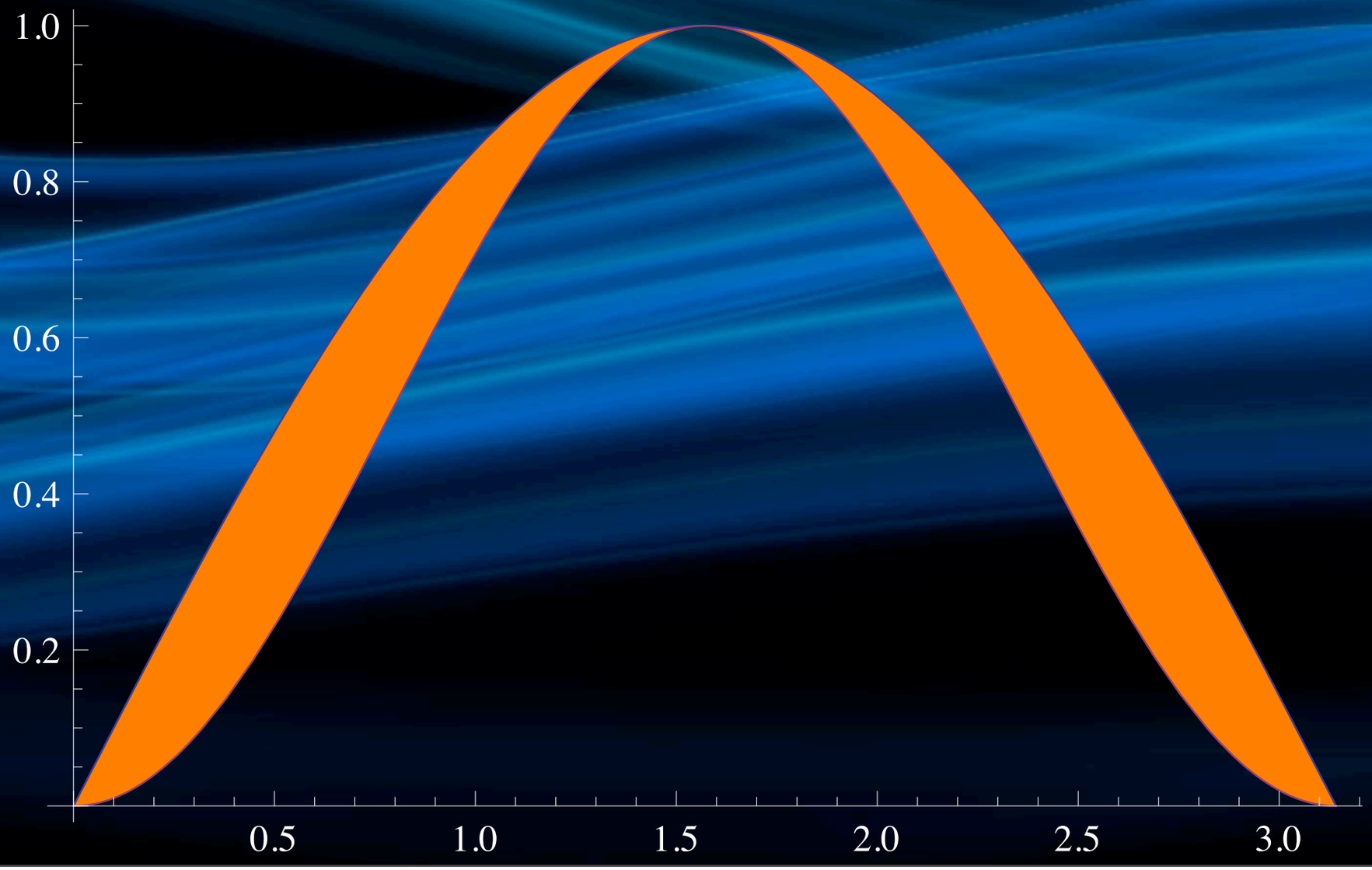
$\cos(x)=(1-u^2)/(1+u^2)$

# Area Computation


g
f
a
b

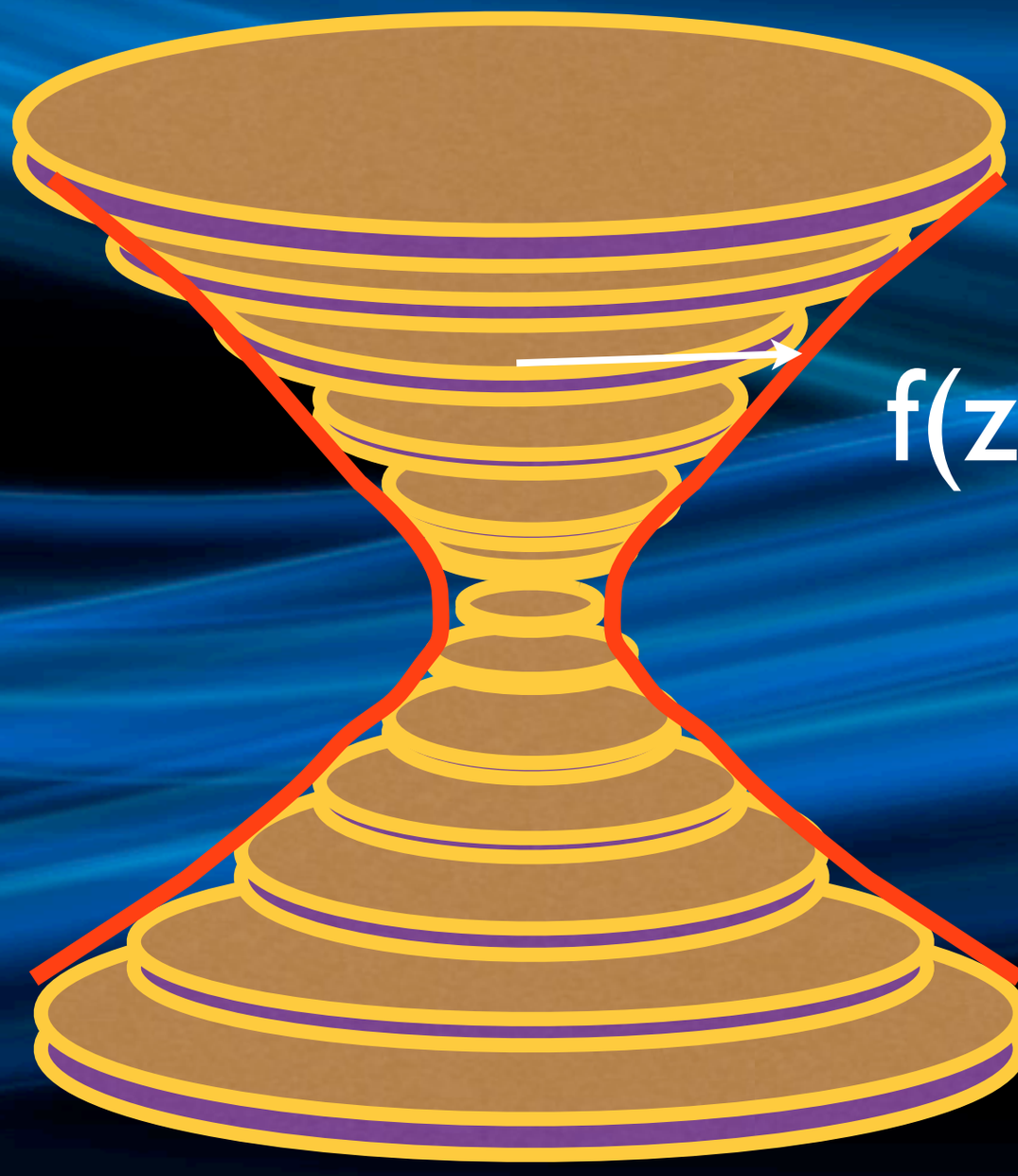
# Problem 12

Find the area enclosed between $\sin(x)$ and $\sin^2(x)$



# Volume Computation

# Problem 13 Find volume



$f(z) = 1+z^2$

$-1<z<1$

cylindrical slices
and shell

1

-1

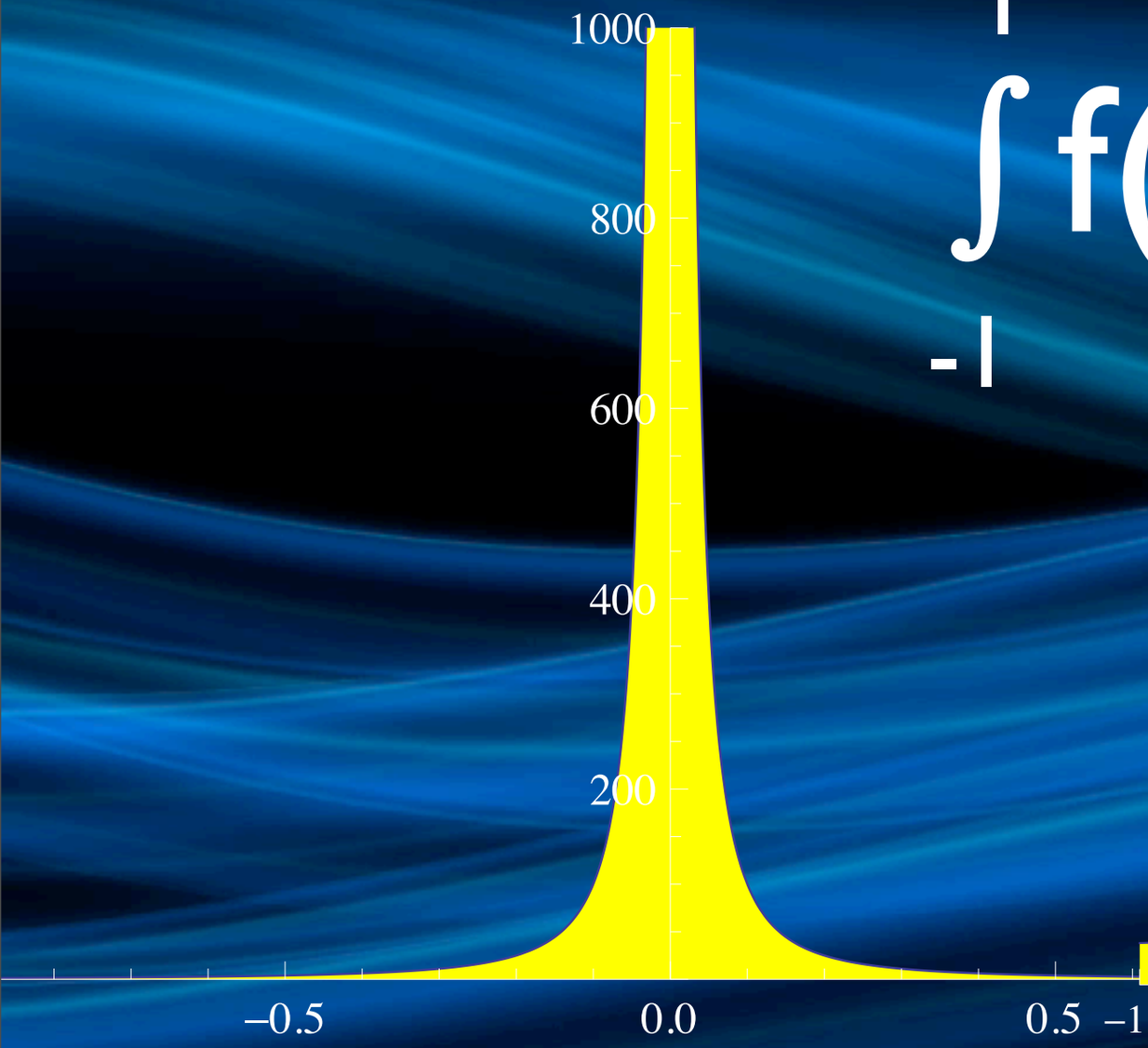
π 56/15

# Improper integrals

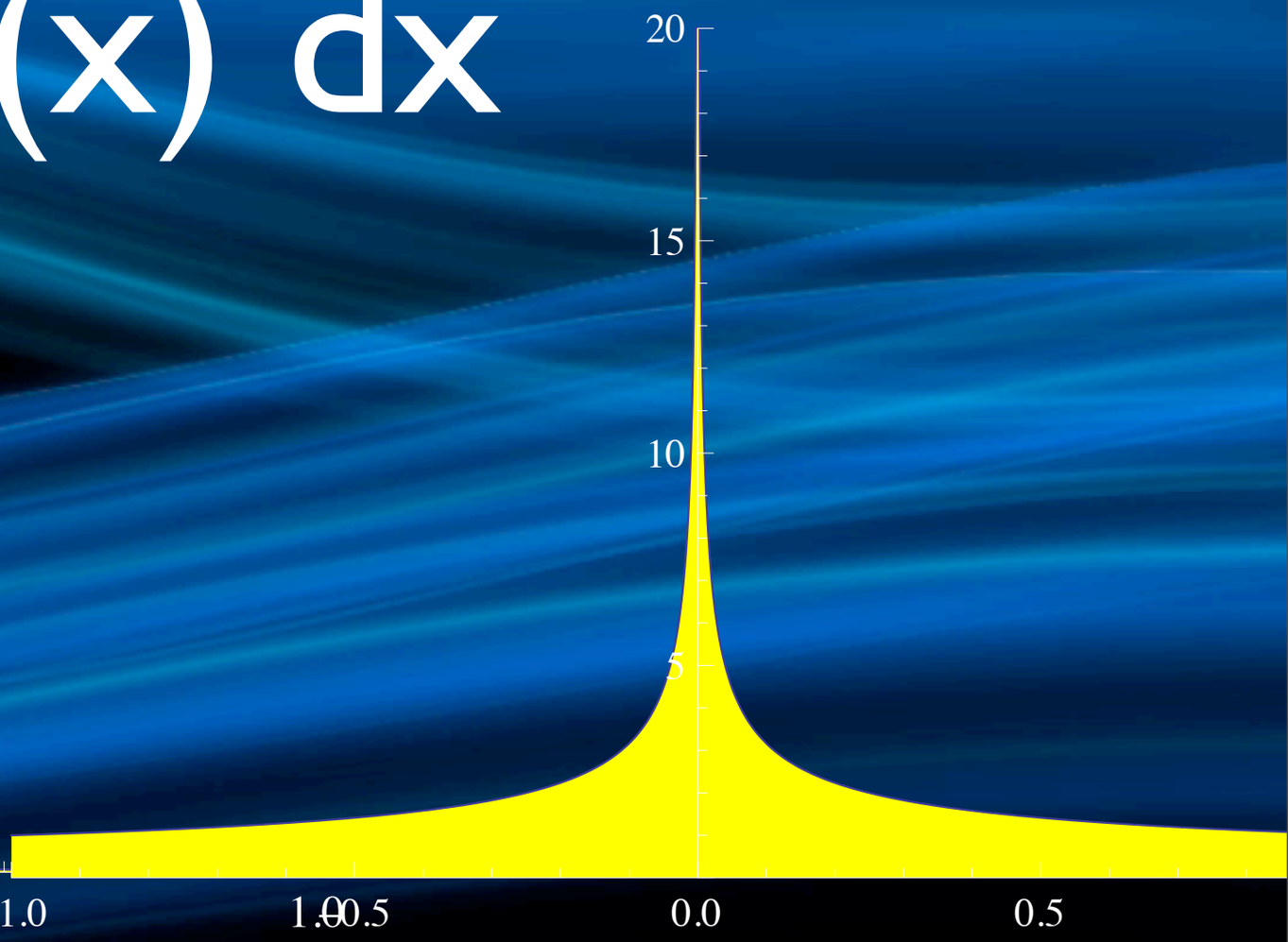
"I love you from here to infinity!"

# Problem 13: Which converges?

$$\int_{-1}^{1} f(x)\,dx$$

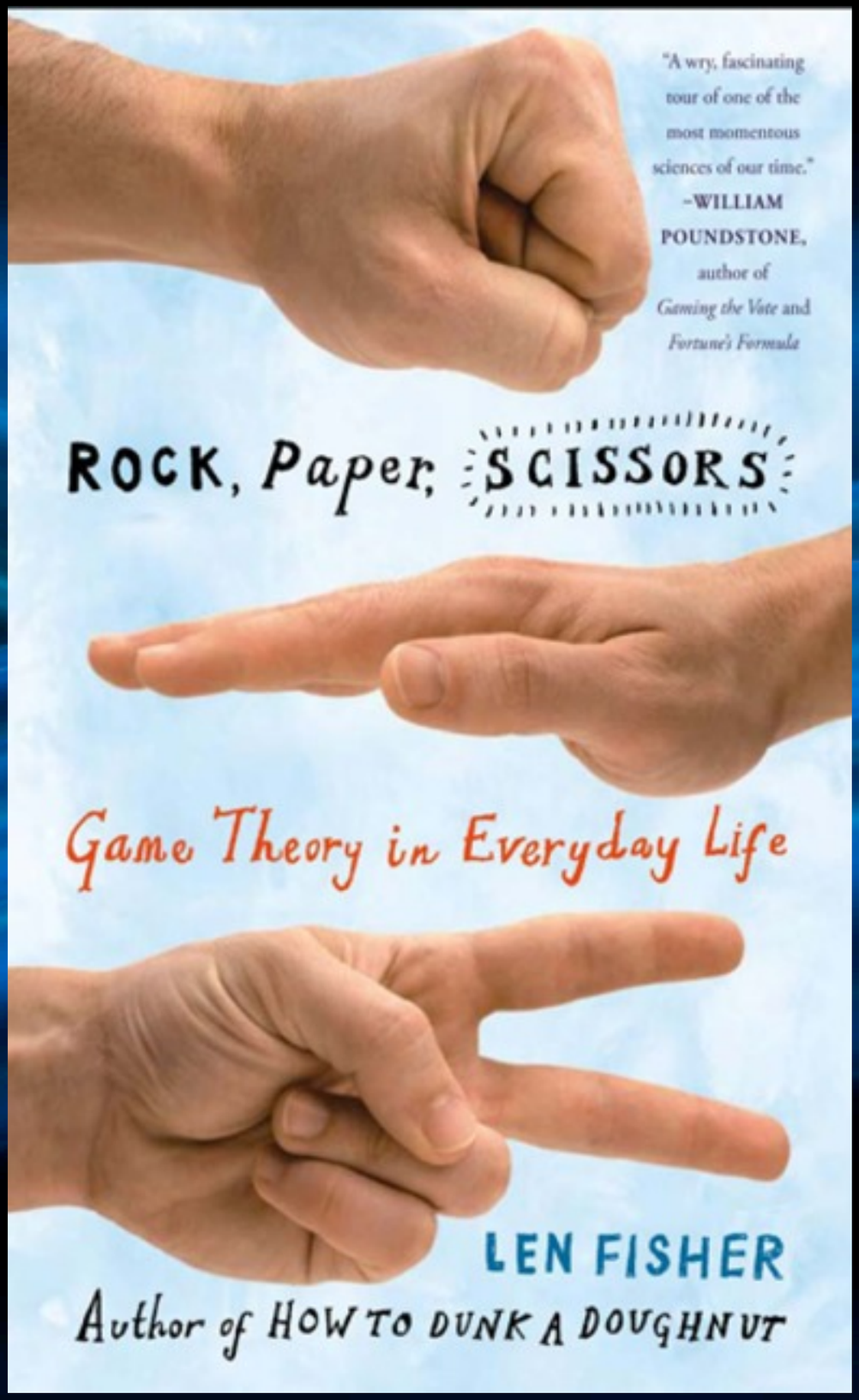



$f(x)=1/x^2$




$f(x)=1/\sqrt{|x|}$

# Book to win


"A wry, fascinating tour of one of the most momentous sciences of our time."
–WILLIAM POUNDSTONE,
author of
Gaming the Vote and
Fortune's Formula
ROCK, Paper, SCISSORS
Game Theory in Everyday Life
LEN FISHER
Author of HOW TO DUNK A DOUGHNUT

Yell the answer:
(you have to give a short explanation)

# No computers or Wolfram alpha!

# No computers or Wolfram alpha!

3

No computers or Wolfram alpha!

$$3 \quad 2$$

No computers or Wolfram alpha!

3 2 1

# Problem 14: What is:

$$\frac{8}{\Gamma(2/7)} \int_0^\infty e^{-x^7} x dx$$

Tuesday, May 10, 2011

# Applications

- Statistics: PDF and CDF
- Business: Marginal, total,average cost
- Physics: position, velocity acceleration, free fall
- Music: hull function, scale
- Computer science: curvature, Chaikin
- Economics: Strawberry theorem
- Gastronomy: wobbly table
- Psychology: perception catastrophe
- Numerics: Simpson, Newton mathod
- Entertainment: Math stories

# Problem 15

$$F(x) = -x^2 \log(x)$$

Verify that $g(x) = F(x)/x$ is extremal at $F' = g$

# Problem 16


A car accelerates with $a(t) = 1+\sin(t)$ It is at $x=0$ at time $t=0$ and has zero velocity then. Where is it at time $t=2\pi$?

# How to build a 1,000mph car

## Hybrid rocketry will be used to drive faster than ever before

May 5th 2011 | from the print edition



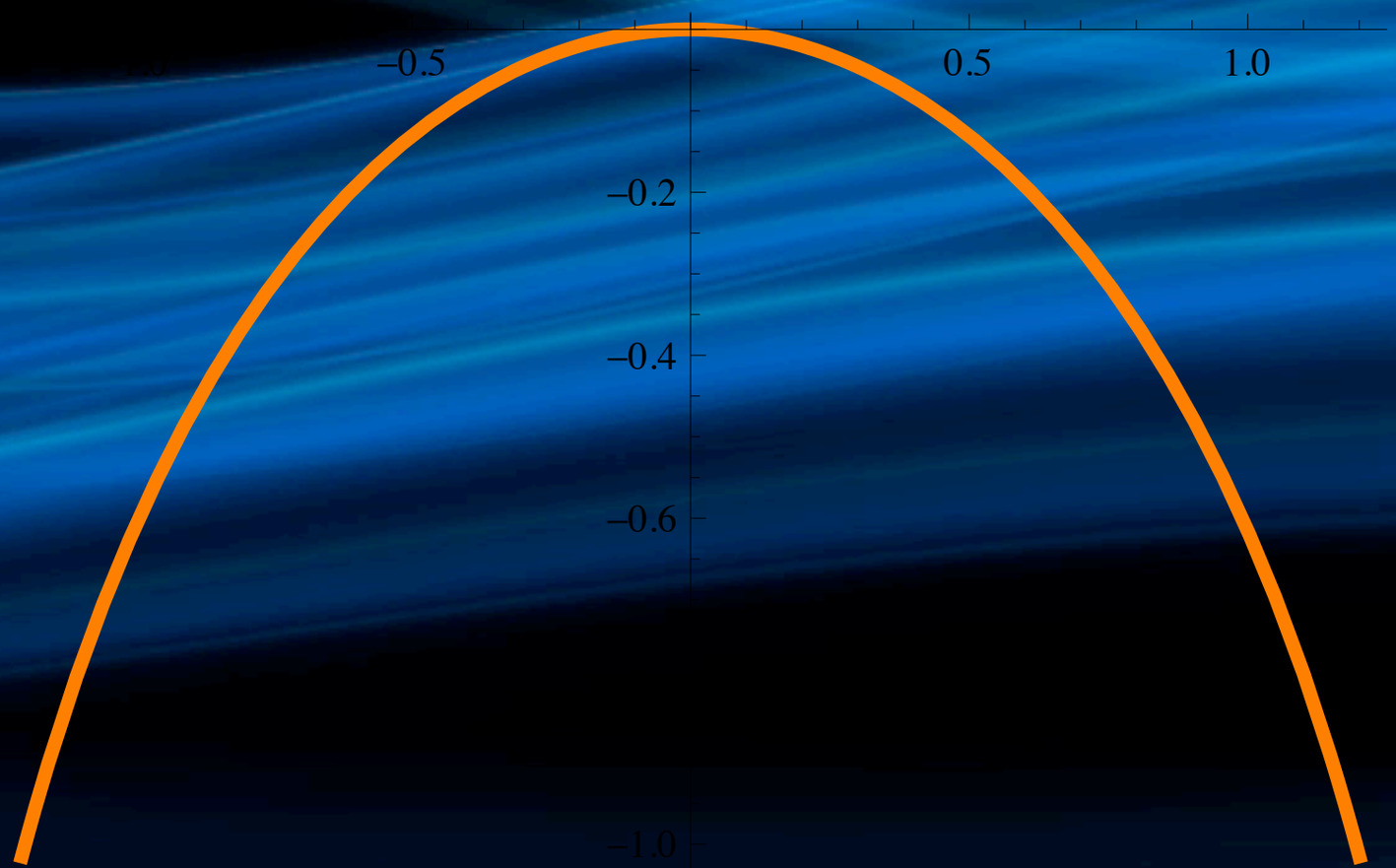

THIS summer Daniel Jubb will perform the equivalent of lighting the blue touch paper and standing clear. The 27-year-old will undertake the first full test firing of a hybrid rocket which he has designed to help a British team set a new land-speed record by driving at 1,000mph (1,609kph). Mr Jubb's rocket, however, will also need the assistance of a powerful EJ200 jet engine from a Typhoon fighter aircraft and a Cosworth Formula 1 racing engine if *Bloodhound SSC* (supersonic car) is to become the fastest thing on wheels.

Combining a rocket, a jet and a racing-car engine into one vehicle is engineering of an extreme sort, but record-breaking often demands that new problems be solved. Mr Jubb's task was to build a rocket that could be used safely in a car, but was also controllable and could be switched off quickly in the event of an emergency.

A rocket works by burning fuel with an

# Problem 17

Find the curvature of log(cos(x)) at x=π/4



# Last Problem 18

# Which function is that?

A $f(x) = \sin(1000\ x)\ \sin(x))$

B $f(x) = \sin(10\ \sin(100x)))$

C $f(x) = \sin(1000\ \sin(x\ *x))$

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A course which fulfills a general education requirement.

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The dissection of life

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See the world through the eyes of a genius (Newton)

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Interesting, tedious and not too bad

The smell of waffles in Annenberg: overwhelming yet delicious

# Nicest application of Calculus?

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The nicest application of calculus is in music

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Stock market and physics of motion

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Economics!

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The derivative of Lady Gaga.

Determine speed at which Oliver can finish a beer at 10 AM.

Strawberries with whipped cream and waffles

# The end