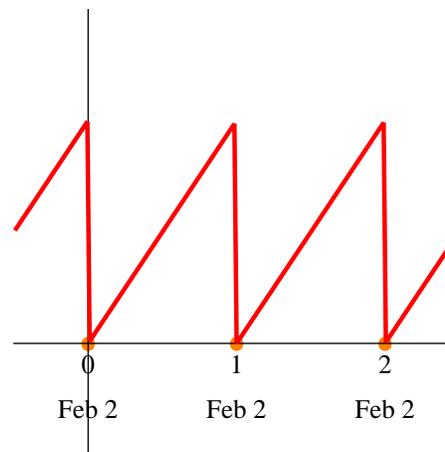


Lecture 5: Worksheet

Its groundhog day and a blizzard is coming. We study extrema and the intermediate value theorem.

The intermediate value theorem

- 1 Today on groundhog day, the average temperature is 33° Fahrenheit. Last summer, there was an average temperature was 77.2° . Was there a time between July 1, 2010 and Feb 2, 2011, when the temperature was exactly 50° ?
- 2 We have got 38 inches of snow this month already. Does this mean there was a time that we had 20 inches of snow on the ground?
- 3 Is there a point x , where $1/\sin(x) = 1/2$? Why does the intermediate value theorem not give such a point? We have $1/\sin(\pi/2) = 1$ and $1/\sin(3\pi/2) = -1$.
- 4 Is there a point, where $\text{sign}(x) = 1/2$? Remember the signum function. It is 1 for positive numbers, 0 for 0 and -1 for negative numbers.
- 5 Lets call the function $f(x) = x - \text{floor}(x)$ the **ground hog function**. If you know the movie with **Bill Murray**, you know why. Can you find an interval on which the intermediate value theorem fails?



The derivative and extrema

- 6 Find a concrete function which has only one local maximum, and no local minimum.
- 7 We have seen a remarkable theorem assuring the existence of maxima and minima. In the classical sense this is not true. We will define critical points as points, where $f'(x) = 0$ and see that for $f(x) = x^3$, the derivative is $3x^2$ which is zero at $x = 0$. Does $f(x)$ have a local maximum or minimum at $x = 0$?