Lecture 6: Worksheet

The exponential function

We illuminate the fundamental theorem for the exponential function $\exp(x) = (1+h)^{x/h}$. While the discussion could be done for any $h > 0$ we look at the special case where $h = 1$ in which case $\exp(x) = 2^x$ maps positive integers to positive integers. You have verified in a homework that

$$D\exp(x) = \exp(x) \ .$$

From the fundamental theorem, we get $SD\exp(x) = S\exp(x) = \exp(x) - \exp(0)$ for integers x. That is

$$S\exp(x) = \exp(x) - 1 \ .$$

In other words, for the exponential function, we know both the derivative and the integral. [1]

1 The formula $S\exp(x) = \exp(x) - 1$ tells for $x = 5$ that $1 + 2 + 4 + 8 + 16 = 32 - 1$. Verify it for $x = 7$.

2 Because $S\exp(x) = \exp(x) - 1$ we can interpret $\exp(x) - 1$ as an area of a union of rectangles. In the picture below, shade an area $\exp(3) - 1$.

3 In the right of the two pictures, there is a line vertical segment which has length $\exp(3)$. Which one?

[1]Later in this course, we will look at these two formulas in the limit $h \to 0$, where

$$\frac{d}{dx}\exp(x) = \exp(x), \qquad \int_0^x \exp(t)\, dt = \exp(x) - 1 \ .$$

4 We know $D\exp(x) = \exp(x)$. Why is also the following formula true?

$$D(\exp(x) - 1) = \exp(x)$$

5 The just verified formula can be interpreted as a difference between areas and so an area. Which one for $x = 4$?