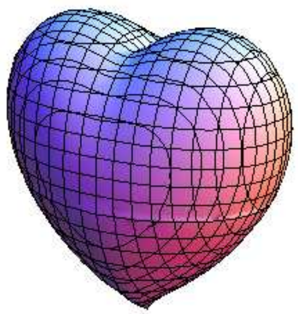
Lecture 10: Worksheet

The chain rule

On this valentine day, we preview a nice application of the chain rule. We will cover this later in the course.



The **Valentine equation** $(x^2+y^2-1)^3 - x^2y^3 = 0$ relates x with y, but we can not write the curve as a graph of a function $y = g(x)$. Extracting y or x is difficult since they are in love. The set of points satisfying the equation looks like a heart. Well, romance is known to be complicated!

You can check that $(1,1)$ satisfies the Valentine equation. Near it, the curve looks like the graph of a function $g(x)$. Lets fill that in and look at the function

$$f(x) = (x^2 + g(x)^2 - 1)^3 - x^2 g(x)^3$$

The key is that $f(x)$ is actually zero and if we take the derivative, then we get zero too. Using the chain rule, we can take the derivative

$$f'(x) = 3(x^2+g(x)^2-1)(2x+2g(x)g'(x)) - 2xg(x)^3 - x^2 3g(x)^2 g'(x) = 0$$

Magically, we can solve for g'

$$g'(x) = -\frac{3(x^2+g(x)^2-1)2x - 2xg(x)^3}{3(x^2+g(x)^2-1)2g(x) - 3x^2 g(x)^2} .$$

Filling in $x = 1, g(x) = 1$, we see this is $-4/3$. We have computed the slope of g without knowing g. Isn't that magic? If this was a bit too complicated, don't worry. We will have an entire lecture on this later in the course.

1 Compute the derivative of $f(x)$ using the chain rule and verify the formula above.