# Lecture 11: Worksheet
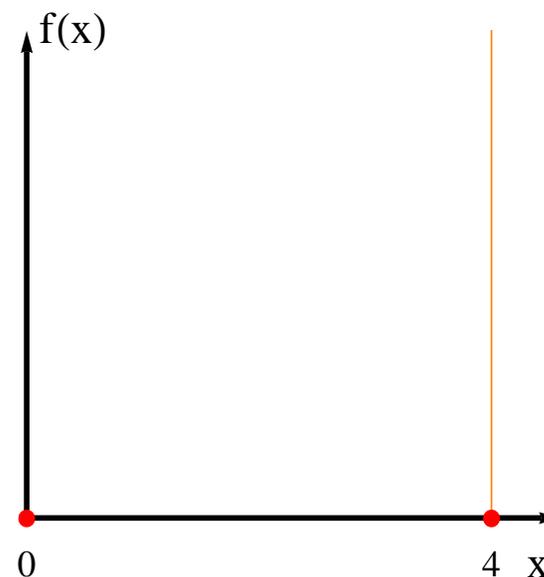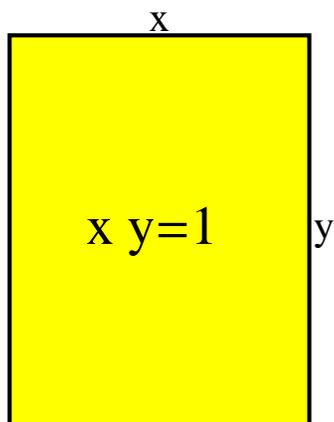
## Critical points and extrema

Which rectangle of fixed area $xy = 1$ has minimal circumference $2x + 2y$?

We have to extremize the function

$$f(x) = 2x + \frac{2}{x} \; .$$

**1** Differentiate the function $f$. For which $x$ is it continuous?

**2** Find the critical points of $f$, the places where $f'(x) = 0$.

**3** Sketch the graph of $f$ on the interval $(0, 4]$.





A related but much more difficult problem is to find the shape with fixed area 1 which has minimal circumference. A more advanced flavor of calculus allows to solve this: the **calculus of variations**.
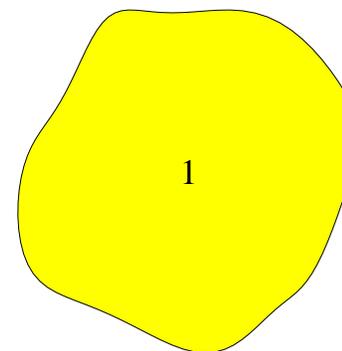


So, which is the winner? You might have guessed. The circle. The rectangle example illustrates already that **symmetry** is often favored in extremization problems.