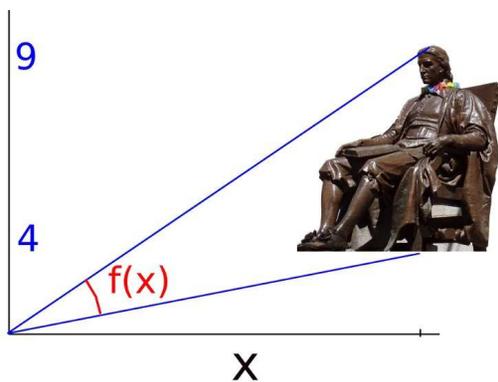


## Lecture 12: Worksheet

### Extrema with boundaries

The following famous problem is usually asked with the Statue of liberty. At Harvard, we of course want to use the John Harvard Statue. It is a common situation. You want to look at a statue. If you are too close below it, the viewing angle becomes small. If you are far away, the viewing angle decreases again. There is an optimal distance where the viewing angle is maximal.



At which distance  $x$  do you see most of the John Harvard Statue?  
Assume the part you want to see 4 to 9 feet higher than your eyes.

- 1 Verify that the angle you see from the statue is

$$f(x) = \arctan\left(\frac{9}{x}\right) - \arctan\left(\frac{4}{x}\right).$$

- 2 Differentiate  $f(x)$  to find the minimum.

- 3 Are there any boundary points or points where  $f$  is not differentiable?

- 4 Find the global maximum of  $f$ .

- 5 Is there a global minimum of  $f$ ?

Here is a graph of part of the function  $f$ .

