Lecture 16: Worksheet

The mean value theorem

In this class, we have at various places looked at calculus with discrete eyes, where

$$Df(x) = [f(x + h) - f(x)]/h .$$

We look here at the question whether there is a discrete version of Rolle's theorem. You may have the opportunity to find a new result. Note that quantum results hold in general for functions which are only continuous. No differentiability is needed.

This worksheet might give you an idea what **research** is about. You do not need the answer yet, whether a result works or not. It is exciting because nobody else does now simply because nobody has studied the question yet!

1 **Quantum Rolle:** Given an interval $[a, b]$ from which we assume that its length is larger than h. Given a continuous function f such that $f(a) = f(b) = 0$. Is it true that there is a point p in that interval for which $Df(p) = 0$? Play and doodle around with examples.

2 **Quantum mean value theorem:** Given an interval $[a, b]$ and a function f. Is it true that there is a point p such that

$$Df(p) = \frac{f(b) - f(a)}{b - a} ?$$

Play around with examples.

3 Argue that you can "tilt" the setting as in the continuum so that if the quantum Rolle result holds, then the quantum mean value theorem holds.