Lecture 27: Review for second midterm

Major points

The **intermediate value theorem** assures that there is $x \in (a,b)$ with $f'(x) = (f(b) - f(a))/(b-a)$. A special case is Rolle's theorem, where $f(b) = f(a)$.

Catastrophes are parameter values where a local minimum disappears. Typically the system jumps then to a lower minimum.

Definite integrals $F(x) = \int_0^x f(x)\ dx$ are defined as a limit of Riemann sums S_n/n.

A function $F(x)$ satisfying $F' = f$ is called the anti-derivative of f. The general anti-derivative is $F + C$ where C is a constant.

The **fundamental theorem of calculus** tells $d/dx \int_0^x f(x)\ dx = f(x)$ and $\int_0^x f'(x)\ dx = f(x) - f(0)$.

The integral $\int_a^b g(x) - f(x)\ dx$ is the **signed area between the graphs** of f and g. Places, where $f < g$ are counted negative.

The integral $\int_a^b A(x)\ dx$ is a **volume** if $A(x)$ is the area of a slice of the solid perpendicular to a point x on an axes.

Write **improper integrals** as limits of definite integrals $\int_1^\infty f(x)\ dx = \lim_{R\to\infty} \int_1^R f(x)\ dx$. We similarly treat points, where f is discontinuous.

Besides **area**, **volume**, **total cost**, or **position**, we can compute **averages**, **inertia** or **work** using integrals.

If x, y are related by $F(x(t), y(t)) = 0$ and $x(t)$ is known we can compute $y'(t)$ using the chain rule. This is **related rates**.

If $f(g(t))$ is known we can compute $g'(x)$ using the chain rule. This works for inverse functions. This is **implicit differentiation**.

To determine the **catastrophes** for a family $f_c(x)$ of functions, determine the critical points in depence of c and find values c, where a critical point changes from a local minimum to a local maximum.

Important integrals

$\cos(x)$	$\sin(x)$.	$\exp(x)$	$\exp(x)$
$\sin(x)$	$-\cos(x)$.	$\log(x)$	$x\log(x) - x$
$\tan(x)$	$1/\cos^2(x)$.	$1/x$	$\log(x)$
$1/(1+x^2)$	$\arctan(x)$.	$-1/(1+x^2)$	$\text{arccot}(x)$.
$1/\sqrt{1-x^2}$	$\arcsin(x)$	$-1/\sqrt{1-x^2}$	$\arccos(x)$

Improper integrals

$\int_1^\infty 1/x^2\ dx$ Prototype of first type improper integral which exists.
$\int_1^\infty 1/x\ dx$ Prototype of first type improper integral which does not exist.
$\int_0^1 1/x\ dx$ Prototype of second type improper integral which does not exist.
$\int_0^1 1/\sqrt{x}\ dx$ Prototype of second type improper integral which does exist.

The fundamental theorem

$\frac{d}{dx} \int_0^x f(x)\ dx = f(x)$

$\int_0^x f'(x)\ dx = f(x) - f(0)$.

This implies

$\int_a^b f'(x)\ dx = f(b) - f(a)$

Without limits of integration, we call $\int f(x)\ dx$ the **anti derivative**. It is defined up to a constant. For example $\int \sin(x)\ dx = -\cos(x) + C$.

Applications

Calculus applies directly if there are situations where one quantity is the derivative of the other.

function	anti derivative
acceleration	velocity
velocity	position
function	area under the graph
length of cross section	area of region
area of cross section	volume of solid
marginal prize	total prize
power	work
probability density function	cumulative distribution function

Tricks

Whenever dealing with an area or volume computation, make a picture.
In related rates problems, make sure you understand what are variables and what are constants.
For volume computations, find the area of the cross section $A(x)$ and integrate.
For area computations find the length of the slice $f(x)$ and integrate.