

Lecture 36: Worksheet

This worksheet was authored by Sofia ¹, an artificial intelligence calculus teacher and student! The bot could also learn even so only in a primitive way. It had to be told "learn: ...". This entire LaTeX file was generated automatically, (except for this introduction section which has, (thanks to **this** parenthesis) become self-aware and so artificially intelligent.)

Derivatives

Differentiate the following functions: Level 1

- 1 a) $f(x) = x \tan(x)$
b) $f(x) = x + \tan(x)$
c) $f(x) = x \log(x)$
d) $f(x) = e^{-x}x$
e) $f(x) = \cos(x)$

¹Written in the academic year 2003/2004, thanks to a grant from the Harvard Provost together with **Johnny Carlsson**, **Andrew Chi** and **Mark Lezama**. Sofia was a chat bot which would use computer algebra systems to solve calculus problems while chatting, similar to Wolfram Alpha now. The later is of course much more sophisticated. Ours was maybe a 25 week * 4 people * 15 hour = half a person-year project

Integrals

Integrate the following functions: Level 1

- 1 a) $f(x) = \frac{1}{x^2} + 1$
b) $f(x) = \sec^2(x)$
c) $f(x) = 1 - \sin(x)$
d) $f(x) = \sec^2(x) + 1$
e) $f(x) = \frac{1}{2\sqrt{x}}$

Derivatives

Differentiate the following functions: Level 2

- 1 a) $f(x) = x^{3/2} \sec(x)$
b) $f(x) = e^{-x}(x + \sin(x))$
c) $f(x) = 0$
d) $f(x) = e^{-x} \sqrt{x}$
e) $f(x) = \frac{e^x}{\log(e^x)}$

Integrals

Integrate the following functions: Level 2

- 1 a) $f(x) = \frac{x^3-3}{x^4}$
b) $f(x) = \frac{3\sqrt{x-3}}{2}$
c) $f(x) = e^x(\cos(x) - \sin(x))$
d) $f(x) = x - x \tan^2(x) - \tan(x)$
e) $f(x) = e^x(\sin(e^x) + e^x \cos(e^x))$

Derivatives

Differentiate the following functions: Level 3

- 1 a) $f(x) = x^2(x - \tan(x))$
b) $f(x) = \sqrt{x} - \log(x) + x \tan(x)$
c) $f(x) = \frac{e^{-x}}{(x-1)x^2}$
d) $f(x) = \frac{e^{-x}(x+\log(x))}{x}$
e) $f(x) = x^3(x + \log(x))$

Integrals

Integrate the following functions: Level 3

- 1 a) $f(x) = -\frac{4x^4+1}{x}$
b) $f(x) = e^{-x}((-x \log(x) + \log(x) + 1) \sin(x) + x \log(x) \cos(x))$
c) $f(x) = e^{-2x}(1 - 2x) - \sec^2(x)$
d) $f(x) = e^x + \sec^2(x)$
e) $f(x) = \frac{(\log(x)-2) \cos(\sqrt{x}) - \sqrt{x} \log(x) \sin(\sqrt{x})}{\sqrt{x} \log^2(x)}$

Derivatives

Differentiate the following functions: Level 4

- 1 a) $f(x) = \frac{\sin^3(x) + \sin(x) + \sin(\sin(x))}{x+1}$
 b) $f(x) = (x^3 - \sqrt{x^3} - x - 4) \csc(x)$
 c) $f(x) = (e^{-3x} - 9x) \cot(x)$
 d) $f(x) = e^{-x}((x-3)(x-2) - \cos(x))$
 e) $f(x) = \frac{\sqrt{e^x - 3x}}{3x}$

Integrals

Integrate the following functions: Level 4

- 1 a) $f(x) = \frac{-\sqrt{x}(x-6) + 2(x-2)x \sin(x) + 2(x-4) \cos(x)}{2x^3}$
 b) $f(x) = x^4 + (x-2)(4x^3 - \cos(x) - \sec^2(x)) - \sin(x) - \tan(x)$
 c) $f(x) = -\frac{e^{\frac{1}{x}}(3 \sec^2(\frac{1}{x}) + (3x+1)x^2 \tan(\frac{1}{x}))}{2 \tan(\log(x)) + \frac{x^7 \sin(\log(x))}{\sqrt{\cos(\log(x))}}}$
 d) $f(x) = -\frac{2 \tan(\log(x)) + \frac{x^7 \sin(\log(x))}{\sqrt{\cos(\log(x))}}}{2x}$
 e) $f(x) = 6 \tan^5(x) \sec^2(x)$

Derivatives

Differentiate the following functions: Level 6

- 1 a) $f(x) = \frac{1}{x^{3/2}} + \frac{(\sqrt{x} + \log(\sqrt{x})) \cos(\sqrt{x}) \cot(\sqrt{x})}{\sqrt{x+3}}$
 b) $f(x) = -(x+3) \sin^{\frac{3}{2}}(x) (\tan(\sin(x)) - \log(\tan(\sin(x))))$
 c) $f(x) = \frac{1}{x^2} - \frac{2x(\frac{x}{\log(x)} - \log(x))}{x+1} + \sin(x)$
 d) $f(x) = x^5 (x^4 - x + e^x - \cos(x)) - x^3$
 e) $f(x) = \sec(x) (\sin^6(x) + \sin(\sin(x)) + \tan(\sin(x))) - e^x$

Integrals

Integrate the following functions: Level 6

- 1 a) $f(x) = \frac{\sec(x) (\sec(x) \log(\tan(x)) (8\sqrt{\tan(x)} - \sin(\sqrt{\tan(x)}) + 3 \sin(3\sqrt{\tan(x)})) - 8\sqrt{\tan(x)} \log^2(\tan(x)))}{8\sqrt{\tan(x)} \log^2(\tan(x))}$
 b) $f(x) = \frac{1}{2} \left(\frac{1}{\sqrt{x}} - \frac{e^{-x} \left(\frac{6}{\log^4(x)} + \frac{1}{(\log(x)-2)^2} \right)}{x} - 2e^{-x} \left(\frac{1}{\log^3(x)} + \frac{1}{2(\log(x)-2)} \right) \right)$
 c) $f(x) = -\frac{2x^5 \log(x) \sec^2(x) + 2x^4 (3 \log(x) - 1) \tan(x) + \log^2(x)}{x^2 \log^2(x)}$
 d) $f(x) = -\frac{5x^{7/2} + 20x^6 - 24x^4 + 2}{2x^2}$
 e) $f(x) = -\frac{2x^2 \sqrt{\cos(x)} + x \log(x) \sin(x) + (6 \log(x) - 2) \cos(x)}{2x^4 \sqrt{\cos(x)}}$