3/1/2011: First hourly Practice

Your Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly and except for multiple choice problems, give computations. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.
- All unspecified functions are assumed to be smooth: one can differentiate arbitrarily.
- The actual exam has a similar format: TF questions, multiple choice and then problems where work needs to be shown.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False questions (20 points) No justifications are needed.

1) T F The function $\cot(x)$ is the inverse of the function $\tan(x)$.

2) T F We have $\cos(x)/\sin(x) = \cot(x)$

3) T F $\sin(3\pi/2) = -1$.

4) T F The function $f(x) = \sin(x)/x$ has a limit at $x = 0$.

5) T F For the function $f(x) = \sin(\sin(\exp(x)))$ the limit $\lim_{h\to 0}[f(x+h)-f(x)]/h$ exists.

6) T F If a differentiable function $f(x)$ satisfies $f'(3) = 3$ and is f' is odd then it has a critical point.

7) T F The l'Hopital rule assures that the derivative satisfies $(f/g)' = f'/g'$.

8) T F The intermediate value theorem assures that a continuous function has a derivative.

9) T F The function $f(x) = (x+1)/(x^2-1)$ is continuous everywhere.

10) T F If f is concave up on $[1, 2]$ and concave down on $[2, 3]$ then 2 is an inflection point.

11) T F There is a function f which has the property that its second derivative f'' is equal to its negative f.

12) T F The function $f(x) = [x]^4 = x(x+h)(x+2h)(x+3h)$ has the property that $Df(x) = 4[x]^3 = 4x(x+h)(x+2h)$, where $Df(x) = [f(x+h) - f(x)]/h$.

13) T F The quotient rule is $d/dx(f/g) = (f'g - fg')/g^2$ and holds whenever $g(x) \neq 0$.

14) T F The chain rule assures that $d/dx f(g(x)) = f'(g(x)) + f(g'(x))$.

15) T F If f and g are differentiable, then $(3f + g)' = 3f' + g'$.

16) T F For any function f, the Newton step $T(x)$ is continuous.

17) T F One can rotate a four legged table on an arbitrary surface such that all four legs are on the ground.

18) T F The fundamental theorem of calculus relates integration S with differentiation D. The result is $DSf(x) = f(x), SDf(x) = f(x) - f(0)$.

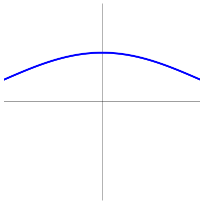
19) T F The product rule implies $d/dx(f(x)g(x)h(x)) = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$.

20) T F Euler and Gauss are the founders of infinitesimal calculus.

Problem 2) Matching problem (10 points) No justifications are needed.

Match the following functions with their graphs.

Function	Fill in 1-8
$x^2 - x$	
$\exp(-x)$	
$\sin(3x)$	
$\log(\|x\|)$	
$\tan(x)$	
$1/(2 + \cos(x))$	
$x - \cos(6x)$	
$\sin(3x)/x$	



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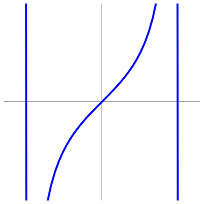

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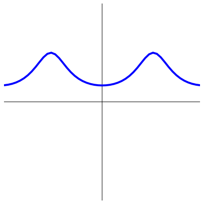

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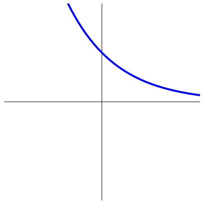

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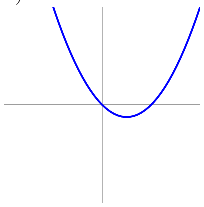

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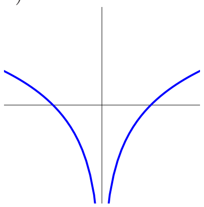

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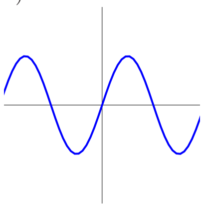

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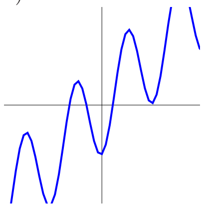

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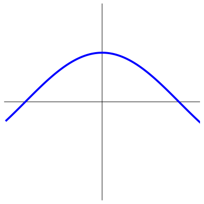
Problem 3) Matching problem (10 points) No justifications are needed.

Match the following functions with their derivatives.

Function	Fill in the numbers 1-8
graph a)	
graph b)	
graph c)	
graph d)	
graph e)	
graph f)	
graph g)	
graph h)	



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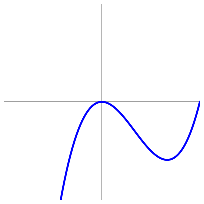

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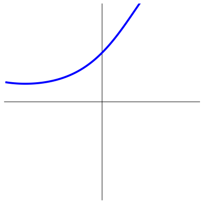

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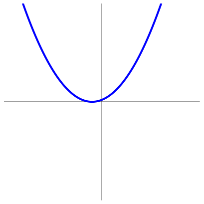

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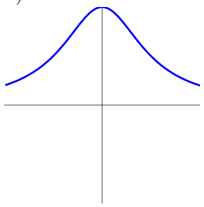

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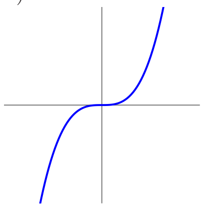

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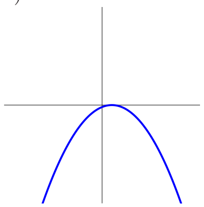

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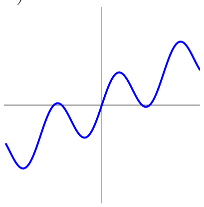

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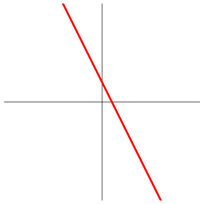

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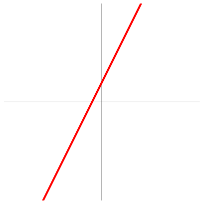

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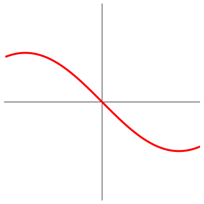

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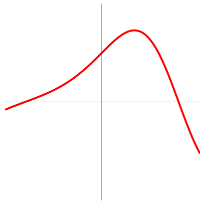

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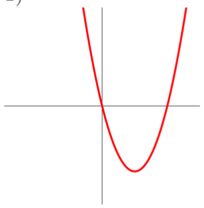

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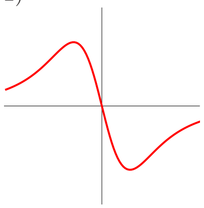

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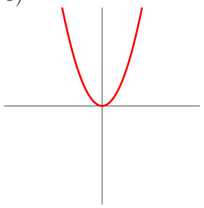

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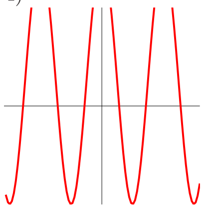

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Problem 4) Functions (10 points) No justifications are needed

Match the following functions. In each of the cases, exactly one of the choices A-C is true.

Function	Choice A	Choice B	Choice C	Enter A-C
$\frac{x^4-1}{x-1}$	$1+x+x^2+x^3$	$1+x+x^2$	$1+x+x^2+x^3+x^4$	
2^x	$e^{2\log(x)}$	$e^{x\log(2)}$	$2^{e\log(x)}$	
$\sin(2x)$	$2\sin(x)\cos(x)$	$\cos^2(x)-\sin^2(x)$	$2\sin(x)$	
$1/x+1/(2x)$	$1/(x+2x)$	$3x/2$	$1/(x+2x)$	
e^{x+2}	$e^x e^2$	$2e^x$	$(e^x)^2$	
$\log(4x)$	$4\log(x)$	$\log(4)\log(x)$	$\log(x)+\log(4)$	
$\sqrt{x}^3$	$x^{3/2}$	$x^{2/3}$	$3\sqrt{x}$	

Problem 5) Roots (10 points)

Find the roots of the following functions

a) (2 points) $7\sin(3\pi x)$

b) (2 points) $x^5 - x$.

c) (2 points) $\log|ex|$.

d) (2 points) $e^{5x} - 1$

e) (2 points) $8x/(x^2+4) - x$.

Problem 6) Derivatives (10 points)

Find the derivatives of the following functions:

a) (2 points) $f(x) = \cos(3x)/\cos(10x)$

b) (2 points) $f(x) = \sin^2(x)\log(1+x^2)$

c) (2 points) $f(x) = 5x^4 - 1/(x^2+1)$

d) (2 points) $f(x) = \tan(x) + 2^x$

e) (2 points) $f(x) = \arccos(x)$

Problem 7) Limits (10 points)

Find the limits $\lim_{x\to 0} f(x)$ of the following functions:

a) (2 points) $f(x) = (x^6 - 3x^2 + 2x)/(1 + x^2 - \cos(x))$.

b) (2 points) $f(x) = (\cos(3x) - 1)/(\cos(7x) - 1)$.

c) (2 points) $f(x) = \tan^3(x)/x^3$.

d) (2 points) $f(x) = \sin(x)\log(x^6)$

e) (2 points) $f(x) = 4x(1-x)/(\cos(x) - 1)$.

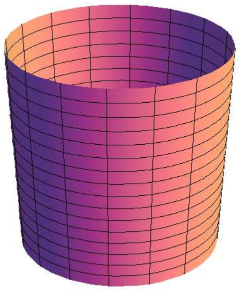
Problem 8) Extrema (10 points)

a) (5 points) Find all local extrema of the function $f(x) = 30x^2 - 5x^3 - 15x^4 + 3x^5$ on the real line.

b) (5 points) Find the global maximum and global minimum of the function $f(x) = \exp(x) - \exp(2x)$ on the interval $[-2, 2]$.

Problem 9) Extrema (10 points)

A cup of height h and radius r has the volume $V = \pi r^2 h$. Its surface area is $\pi r^2 + \pi r h$. Among all cups with volume $V = \pi$ find the one which has minimal surface area. Find the global minimum.



Problem 10) Newton method (10 points)

a) (3 points) Produce the first Newton step for the function $f(x) = e^x - x$ at the point $x = 1$.

b) (4 points) Produce a second Newton step.

c) (3 points) Find the Newton step map $T(x)$ if the function $f(x)$ is replaced by the function $3f(x)$.