Lecture 5: Worksheet

Groundhog day is tomorrow. We study extrema and the intermediate value theorem.

The intermediate value theorem

1 Today the average temperature is 38° Fahrenheit. Last year at this time, the temperature had been 33°. Was there a time between Feb 1, 2011 and Feb 1, 2012, when the temperature was exactly 35°?

2 Is there a point x, where

$$\frac{1}{\sin(x)} = \frac{1}{2} \; .$$

Why does the intermediate value theorem not give such a point? We have $1/\sin(\pi/2) = 1$ and $1/\sin(3\pi/2) = -1$.

3 The earth's diameter is 12'756 km in average. Is there a point on earth where the distance to its antipiode is exactly 12'756 km?

4 The function $f(x) = x - \text{floor}(x)$ is called the **ground hog function**. If you know the movie with **Bill Murray**, you know why. Find an interval where the intermediate value theorem fails.

Feb 2   Feb 2   Feb 2
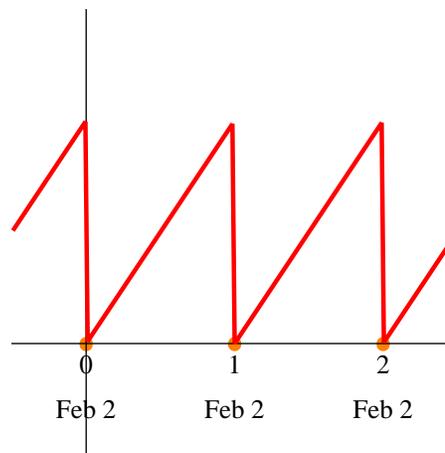
The derivative and extrema

5 Find a concrete function which has only one local maximum, and no local minimum.

6 We have seen a remarkable theorem assuring the existence of maxima and minima. In the classical sense this is not true. We will define critical points as points, where $f'(x) = 0$ and see that for $f(x) = x^3$, the derivative is $3x^2$ which is zero at $x = 0$. Does $f(x)$ have a local maximum or minimum at $x = 0$?