

5/11/2013: Practice final B

Your Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- Except for multiple choice problems, give computations.
- No notes, books, calculators, computers, or other electronic aids are allowed.
- You have 180 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
12		10
13		10
Total:		140

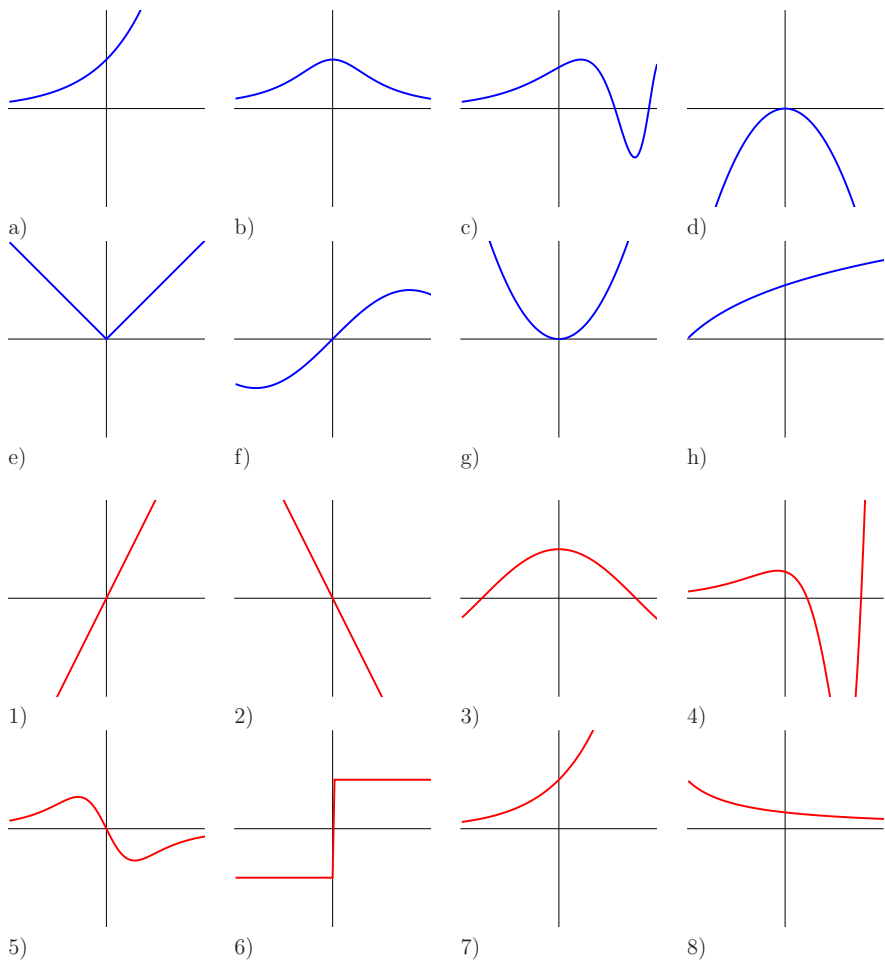
Problem 1) TF questions (20 points) No justifications are needed.

- 1) ☐ T ☐ F The quantum exponential function $\exp_h(x) = (1 + h)^{x/h}$ satisfies $D \exp_h(x) = \exp_h(x)$ for $h > 0$.
- 2) ☐ T ☐ F The function $\text{sinc}(x) = \sin(x)/x$ has a critical point at $x = 0$.
- 3) ☐ T ☐ F The limit of $1/\log(1/|x|)$ for $x \rightarrow 0$ exists.
- 4) ☐ T ☐ F The strawberry theorem tells that for any $f(x)$, its anti-derivative $F(x)$ and $g(x) = F(x)/x$ the points $f = g$ are the points where $g' = 0$.
- 5) ☐ T ☐ F The function $f(x) = \tan(x)$ has a vertical asymptote at $x = \pi/2$.
- 6) ☐ T ☐ F The function $x/(1 + x)$ converges to 1 for $x \rightarrow \infty$ and has therefore a horizontal asymptote.
- 7) ☐ T ☐ F The function $f(x) = \tan(x)$ is odd: it satisfies $f(x) = -f(-x)$.
- 8) ☐ T ☐ F The function $\sin^3(x)/x^2$ is continuous on the real line.
- 9) ☐ T ☐ F With $Df(x) = f(x+1) - f(x)$ we have $D(fg)(x) = Dfg(x+1) + f(x)Dg(x)$.
- 10) ☐ T ☐ F If f is continuous and has a critical point a for h , then f has either a local maximum or local minimum.
- 11) ☐ T ☐ F The limit of $[\frac{1}{3+h} - \frac{1}{3}]/h$ for $h \rightarrow 0$ is $-1/9$.
- 12) ☐ T ☐ F The function $(\cos(x) + \sin(3x))/(\sin(4x) + \cos(3x))$ can be integrated using trig substitution.
- 13) ☐ T ☐ F The marginal cost is the anti-derivative of the total cost.
- 14) ☐ T ☐ F The cumulative distribution function is the anti-derivative of the probability density function.
- 15) ☐ T ☐ F The function $\sqrt{1 - x^2}$ can be integrated by a trig substitution $x = \cos(u)$.
- 16) ☐ T ☐ F The integral $\int_0^1 1/x^2 dx$ is finite.
- 17) ☐ T ☐ F The chain rule tells that $d/dx f(g(x)) = f'(x)g'(x)$.
- 18) ☐ T ☐ F For the function $f(x) = \sin(100x)$ the hull function is constant.
- 19) ☐ T ☐ F The trapezoid method is also called Simpson rule.
- 20) ☐ T ☐ F If $f''(x) > 0$, then the curvature of f is positive.

Problem 2) Matching problem (10 points) No justifications are needed.

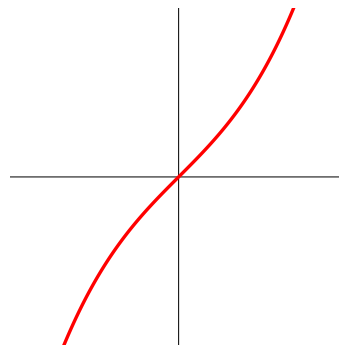
Match the following functions with their derivatives.

Function	Fill in the numbers 1-8
graph a)	
graph b)	
graph c)	
graph d)	
graph e)	
graph f)	
graph g)	
graph h)	



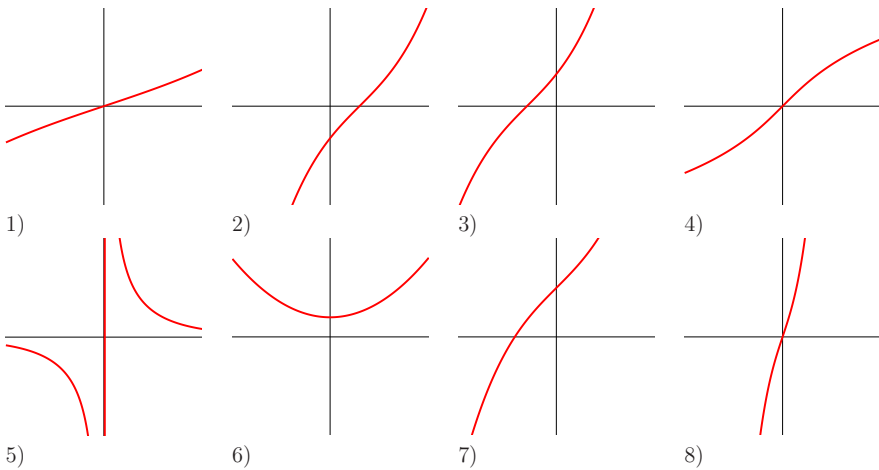
Problem 3) Matching problem (10 points) No justifications are needed.

Here is the graph of a function $f(x)$. Match the following modifications



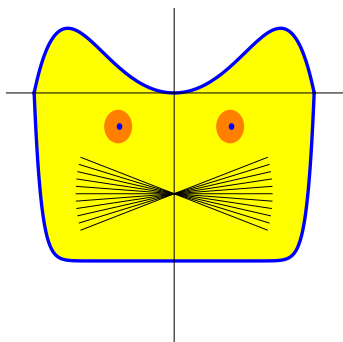
Match the following functions with their graphs.

Function	Fill in 1)-8)
$f(x - 1)$	
$f'(x)$	
$f(x + 1)$	
$f^{-1}(x)$	
$f(x/2)$	
$f(3x)$	
$1/f(x)$	
$f(x) + 1$	



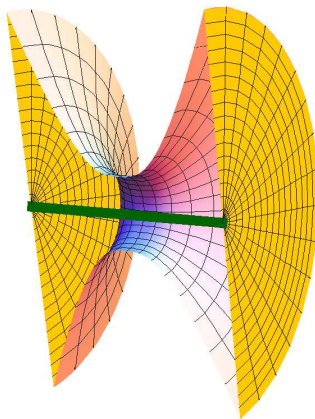
Problem 4) Area computation (10 points)

Find the area of the **cat region** which is the region enclosed by the functions $f(x) = x^{20} - 1$ and $g(x) = x^2 - x^6$. No need to count in the whiskers.



Problem 5) Volume computation (10 points)

We spin the graph of the function $f(x) = \sqrt{1 + |x|^3}$ around the x axes and get a solid of revolution. What is the volume of this solid enclosed between $x = -3$ and $x = 3$? The picture shows half of this solid.



Problem 6) Definite integrals (10 points)

Evaluate the following definite integrals

a) $\int_{-1}^1 \frac{1}{1+x^2} dx$

b) $\int_1^2 x^2 + \sqrt{x} dx$

c) $\int_0^{\sqrt{\pi}} \sin(x^2) 2x dx$.

d) $\int_0^1 \log(4+x) dx$.

Problem 7) Extrema (10 points)

a) (7 points) Analyse the local extrema of the function

$$f(x) = \frac{x}{1+x^2}$$

on the real axes using the second derivative test.

b) (3 points) Are there any global extrema?

Problem 8) Integration by parts (10 points)

a) (5 points) Find the anti-derivative of

$$f(x) = \sin(4x) \cos(3x) .$$

b) (5 points) Find the anti-derivative of

$$f(x) = (x-1)^2 \sin(1+x) .$$

Problem 9) Substitution (10 points)

a) (3 points) Find the integral $\int 3x\sqrt{5x^2-5} dx$.

b) (3 points) What is the anti-derivative of $\int \exp(x^2 - x)(4x - 2) dx$?

c) (4 points) Evaluate the definite integral

$$\int_0^{\pi/2} \sqrt{1 - \cos(x)} \sin(x) dx .$$

Problem 10) Partial fractions, Trig substitution (10 points)

a) Solve the integral

$$\int \frac{2 - x + x^2}{(1 - x)(1 + x^2)}$$

by writing

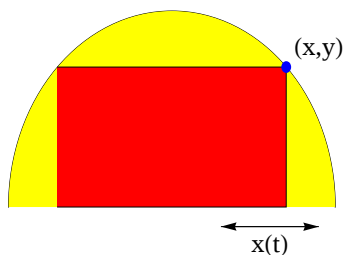
$$\frac{2 - x + x^2}{(1 - x)(1 + x^2)} = \frac{A}{1 - x} + \frac{B}{1 + x^2}.$$

b) Evaluate the integral $\int \sqrt{1 - x^2} dx$.

Problem 11) Related rates (10 points)

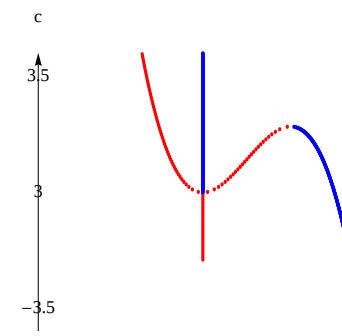
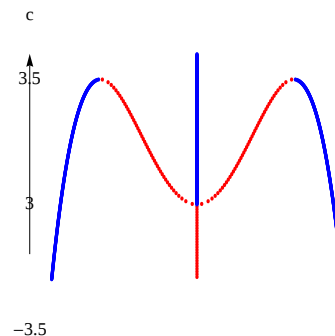
a) (7 points) A rectangle with corners at $(-x, 0)$, $(x, 0)$, (x, y) , $(-x, y)$ is inscribed in a half circle $x^2 + y^2 = 1$ where $y \geq 0$ is in the upper half plane. Assume we move x as $x(t) = t^2$. Find the rate of change of $y(t)$.

b) (3 points) Find the rate of change of the area $A(t) = 2x(t)y(t)$ of the rectangle.



Problem 12) Catastrophes (10 points)

The following two pictures show bifurcation diagrams. The vertical axes is the deformation parameter c . On the left hand side, we see the bifurcation diagram of the function $f(x) = x^6 - x^4 + cx^2$, on the right hand side the bifurcation diagram of the function $f(x) = x^5 - x^4 + cx^2$. As done in class and homework, the bolder continuously drawn graphs show the motion of the local minima and the lighter dotted lines show the motion of the local maxima. In both cases, determine all the bifurcation parameters which are visible. (There are two in both cases).



Problem 13) Applications (10 points)

The **Laplace distribution** is a distribution on the entire real line which has the probability density $f(x) = e^{-|x|}/2$. The variance of this distribution is the integral

$$\int_{-\infty}^{\infty} x^2 f(x) dx.$$

Find it.

