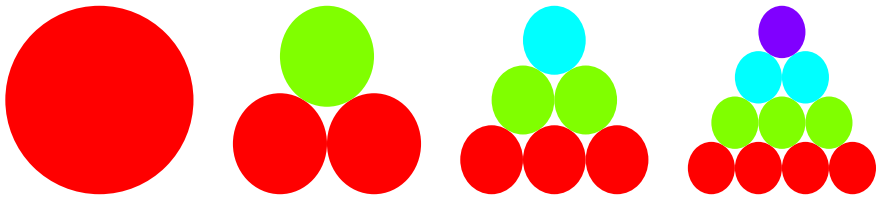
Lecture 1: Worksheet

Triangular numbers

We stack disks onto each other building n layers and count the number of discs. The number sequence we get are called **triangular numbers**.

1	3	6	10	15	21	36	45	...



n=1 n=2 n=3 n=4

This sequence defines a **function** on the natural numbers. For example, $f(4) = 10$.

1 Verify that

$$f(n) = \frac{n(n+1)}{2}$$

gives the above numbers. Check this by algebraically evaluating

$$f(n) - f(n-1) = n \ .$$

Carl-Friedrich Gauss, 1777-1855

Find the next number

Can you find, how the following sequence

$$0, 6, 24, 60, 120, 210, 336, 504...$$

continues? To do so, look at this sequence as a function $f(1) = 0, f(2) = 6, \ldots, f(8) = 504$ and now compute differences. Then use this to go backwards to find the next term $f(9)$.

Tetrahedral numbers

We stack now spheres onto each other building n layers and count the number of spheres. The number sequence we get are called **tetrahedral numbers**.

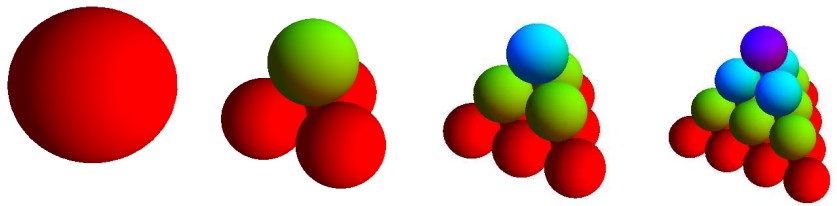
1	4	10	20	35	56	84	120	...

Also this sequence defines a **function**.

2 Verify that

$$g(n) = \frac{n(n+1)(n+2)}{6}$$

satisfies $g(n) - g(n-1) = n(n+1)/2$. We have $g(1) = 1, g(2) = 4, g(3) = 10$.



n=1 n=2 n=3 n=4