Lecture 4: Worksheet

Whats good and whats bad?

Oscillation, poles and jumps are the perils for continuity. In general, we do not have to worry about continuity. There are very few mechanisms which bring you in peril. A function can either start to oscillate like mad, rush to infinity or jump. All cases are usually due to division by zero somewhere.

Good Guys	**Bad Guys**
x^2+4x+6	$1/x$ at 0
$\sin(x), \cos(x)$	$\tan(x)$ at $\pi/2$
$\exp(x)$	$\log\|x\|$ at 0
$\mathrm{sinc}(x) = \frac{\sin(x)}{x}$	$\sec(x) = \frac{1}{\cos(x)}$ at $\pi/2$

Surprises

$\sin(x)/x$	is continuous at 0
$1/\log\|x\|$	is continuous at 0
$x\sin(1/x)$	is continuous at 0

Which functions are continuous?

Which of the following functions are continuous?

1. Is $f(x) = \log 1 + |x|$ continuous at $x = 0$?
2. Is $f(x) = \sqrt{|x|}$ continuous at $x = 0$?
3. Is $f(x) = \frac{1}{\sqrt{|x|}}$ continuous at $x = 0$?
4. Is $\frac{1}{\log|1/x|}$ continuous at $x = 0$?
5. Is $\log(\log|x|)$ continuous at $x = 0$?
6. Is $1/(1+|x|)$ continuous everywhere?

Enemy of continuity

Oscillations, escape to infinity and jumps are reasons for discontinuity.