

## Lecture 5: Worksheet

Groundhog day was on Feb 2, 2-13. Punxsatawney Phil did not see any shadow so that spring is around the corner. We study here extrema and the intermediate value theorem.

### The intermediate value theorem

1 Today the average temperature is  $27^\circ$  Fahrenheit. Argue that there had been a moment this fall/winter where the temperature had been exactly 30 degree Fahrneheit.

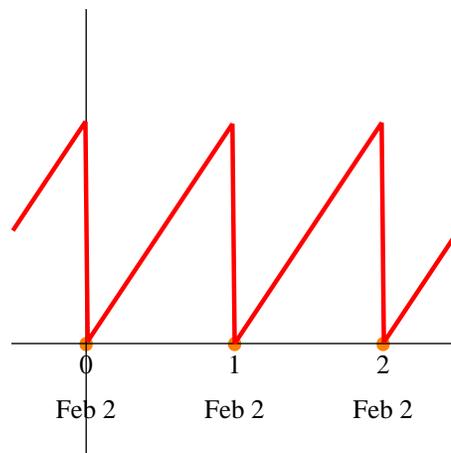
2 Is there a point  $x$ , where

$$\frac{1}{\sin(x)} = \frac{1}{2}.$$

Why does the intermediate value theorem not give such a point? We have  $1/\sin(\pi/2) = 1$  and  $1/\sin(3\pi/2) = -1$ .

3 The earth's diameter is 12'756 km in average. Is there a point on earth where the distance to its antipode is exactly 12'756 km?

4 The function  $f(x) = x - \text{floor}(x)$  is called the **ground hog function**. If you know the movie with **Bill Murray**, you know why. Find an interval where the intermediate value theorem fails.



### The derivative and extrema

5 Find a concrete function which has only one local maximum, and no local minimum.

6 We have seen Fermat's theorem assuring the existence of maxima and minima. In the classical sense this is not true. We will define critical points as points, where  $f'(x) = 0$  and see that for  $f(x) = x^3$ , the derivative is  $3x^2$  which is zero at  $x = 0$ . Does the function  $x^3$  have a local maximum or minimum at  $x = 0$ ?