Lecture 9: Worksheet

The product rule

We practice the product, reciprocal and quotient rule

1 What is the slope of the graph of the function $f(x) = xe^{-x}$ at $x = 0$?

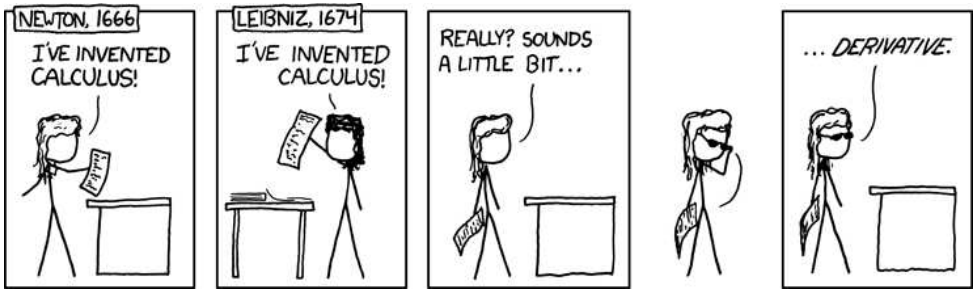
2 Find the derivative of the sinc-function $\sin(x)/x$ at the point $x = 0$.

3 Find the derivative of $\sqrt{x}/x$ at $x = 1$.

4 Find the derivative of $1/e^x$ at $x = 1$.

5 Assume we remember the formula $\sin(2x) = 2\sin(x)\cos(x)$. Differentiate both sides to get a formula for $\cos(2x)$.

6 Find the derivative of $x - 1/(x^2 + 1)$ at $x = 0$.



Source: XKCD

I.

NOVA METHODUS PRO MAXIMIS ET MINIMIS, ITEMQUE TANGENTIBUS, QUAE NEC FRACTAS NEC IRRATIONALES QUANTITATES MORATUR, ET SINGULARE PRO ILLIS CALCULI GENUS *).

Sit (fig. 111) axis AX, et curvae plures, ut VV, WW, YY, ZZ, quarum ordinatae ad axem normales, VX, WX, YX, ZX, quae vocentur respective v, w, y, x, et ipsa AX, abscissa ab axe, vocetur x. Tangentes sint VB, WC, YD, ZE, axi occurrentes respective in punctis B, C, D, E. Jam recta aliqua pro arbitrio assumta vocetur dx, et recta, quae sit ad dx, ut v (vel w, vel y, vel z) est ad XB (vel XC, vel XD, vel XE) vocetur dv (vel dw, vel dy, vel dz) sive differentia ipsarum v (vel ipsarum w, vel y, vel z). His positis, calculi regulae erunt tales.

Sit a quantitas data constans, erit da aequalis 0, et $\overline{da x}$ erit aequalis adx. Si sit y aequ. v (seu ordinata quaevis curvae YY aequalis cuivis ordinatae respondenti curvae VV) erit dy aequ. dv. Jam *Additio et Subtractio*: si sit $z - y + w + x$ aequ. v, erit $\overline{dz - y + w + x}$ seu dv aequ. $dz - dy + dw + dx$. *Multiplicatio*: $d\overline{xv}$ aequ. $xdv + vdx$, seu posito y aequ. xv, fiet dy aequ. $xdv + vdx$. In arbitrio enim est vel formulam, ut xv, vel compendio pro ea literam, ut y, adhibere. Notandum, et x et dx eodem modo in hoc calculo tractari, ut y et dy, vel aliam literam indeterminatam cum sua differentiali. Notandum etiam, non dari semper regressum a differentiali Aequatione, nisi cum quadam cautione, de quo alibi. Porro *Divisio*: $d\frac{v}{y}$ vel (posito z aequ. $\frac{v}{y}$) dz aequ. $\frac{\pm vdy \mp ydv}{yy}$.

Quoad *Signa* hoc probe notandum, cum in calculo pro litera substituitur simpliciter ejus differentialis, servari quidem eadem signa, et pro + z scribi + dz, pro − z scribi − dz, ut ex addi-

*) Act. Erud. Lips. an. 1684.

Leibniz 1684 paper. The product and quotient rule is introduced.