Lecture 10: Worksheet

The chain rule

Remember that the derivative of $f(g(x)$ is $f'(g(x))g'(x)$. This is called the **chain rule**.

For example, the derivative of $\sin(\log(x))$ is $\cos(\log(x))/x$.

We have also seen that we can compute the derivative of inverse functions using the chain rule.

For example, for $f(x) = \log(x)$ we have $\exp(f(x)) = x$ so that the chain rule gives $\exp(f(x))f'(x) = 1$ and so $f'(x) = 1/\exp(f(x)) = 1/x$. Lets practice this.

1 Find the derivative of $\sin^2(x)$ using the product rule.

2 Find the derivative of $\sin^2(x)$ using the chain rule.

3 Find the derivative of $\tan(\sin(x))$.

4 Find the derivative of $\sin(\cos(\exp(x)))$.

5 Find the derivative of $\arcsin(x)$ using the chain rule.

A lovely application of the chain rule

The **Valentine equation** $(x^2 + y^2 - 1)^3 - x^2y^3 = 0$ relates x with y, but we can not write the curve as a graph of a function $y = g(x)$. Extracting y or x is difficult. The set of points satisfying the equation looks like a heart. You can check that $(1, 1)$ satisfies the Valentine equation. Near it, the curve looks like the graph of a function $g(x)$. Lets fill that in and look at the function

$$f(x) = (x^2 + g(x)^2 - 1)^3 - x^2g(x)^3$$

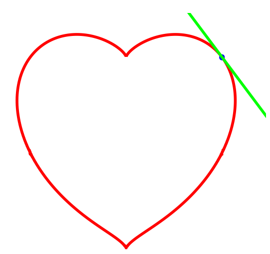
The key is that $f(x)$ is actually zero and if we take the derivative, then we get zero too. Using the chain rule, we can take the derivative

$$f'(x) = 3(x^2+g(x)^2-1)(2x+2g(x)g'(x))-2xg(x)^3-x^23g(x)^2g'(x) = 0$$

We can now solve solve for g'

$$g'(x) = -\frac{3(x^2 + g(x)^2 - 1)2x - 2xg(x)^3}{3(x^2 + g(x)^2 - 1)2g(x) - 3x^2g(x)^2} .$$

Filling in $x = 1, g(x) = 1$, we see this is $-4/3$. We have computed the slope of g without knowing g. Magic!



We will come back to this application of the chain rule later in the course.