4/11/2013: Second midterm practice D

Your Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- Except for multiple choice problems, give computations.
- No notes, books, calculators, computers, or other electronic aids are allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) TF questions (20 points) No justifications are needed.

1) T F The anti derivative of $\log(3x)$ is $x\log(3x) - 3x + C$.

2) T F The fundamental theorem of calculus assures that $\int_a^b f'(x)\,dx = f(a) - f(b)$.

3) T F If $\int_0^x f(t)\,dt$ is monotonically increasing in x for $0 \leq x \leq 1$, then $f(x) \geq 0$ on $0 \leq x \leq 1$.

4) T F The volume of a cone of base radius 1 and height 1 is given by the integral $\int_0^1 \pi x^2\,dx$.

5) T F The mean value theorem assures that if you run from Cambridge to Boston with a $10 Miles/hour$ average, then there was moment along the run, where the velocity was exactly $10 Miles/hour$.

6) T F For any continuous function f, the integral $\int_a^b f(x)\,dx$ is the area under a curve and therefore always positive or zero.

7) T F The sum $(1/n^2 + (2/n)^2 + ... + ((n-1)/n))^2)/n$ is a Riemann sum approximation to $\int_0^1 x^2\,dx$.

8) T F If a differentiable function f has a critical point at 1, then the function $F(x) = \int_0^x f(t)\,dt$ has an inflection point at 1.

9) T F The derivative of the anti derivative of a differentiable function f is always the same as a particular anti derivative of the derivative of f.

10) T F If f is constant 1 then $\int_a^b f(x)\,dx$ is the length of the interval $[a, b]$.

11) T F Subtract the volume of a cone C from a cylinder Z with the same circular base of radius 1 and height 1 and you get the volume of a half sphere of radius 1.

12) T F If $x + y = 10$ is constant and $x' = 3$ then $y' = -3$.

13) T F The identity $\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$ is always true for continuous functions f.

14) T F If $f(x)$ approaches zero for $x \to \infty$, then $\int_1^\infty f(x)\,dx$ is finite.

15) T F If $f_c(x)$ has a minimum x_c which is present for $c < 0$ and disappears for $c > 0$, then $c = 0$ is a catastrophe.

16) T F An indefinite integral is an improper integral which converges.

17) T F Rolle's theorem assures that if $f(a) = f(b) = 0$ then f' has a root inside the interval (a, b).

18) T F The sum $\frac{1}{n}\sum_{k=0}^{n-1}\sin(k/n)$ is a Riemann sum to the integral $\int_0^1 \sin(x)\,dx$.

19) T F The derivative of $\tan(x)$ is $1/(1 + x^2)$.

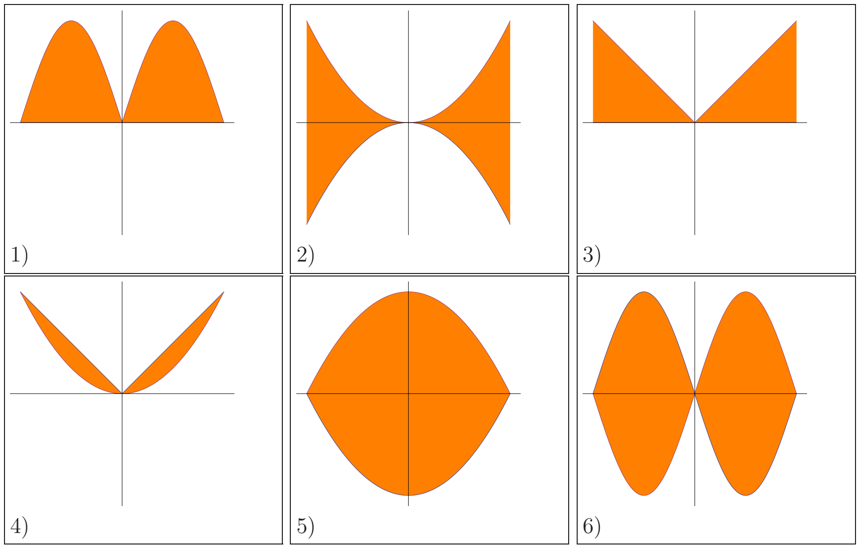
20) T F There are continuous functions for which the anti derivative can not be expressed using known elementary functions.

Problem 2) Matching problem (10 points) No justifications are needed.

a) (6 points) Match the following integrals with the graphs and areas. Each integral matches exactly one area.

Integral	Enter 1-6
$\int_{-1}^{1} \lvert x\rvert \, dx$	
$\int_{-1}^{1} \lvert\sin(\pi x)\rvert \, dx$	
$\int_{-1}^{1} \lvert\sin(\pi x) - (-\sin(\pi x))\rvert \, dx$	

Integral	Enter 1-6
$\int_{-1}^{1} 2 - 2x^2 \, dx$	
$\int_{-1}^{1} 2x^2 \, dx$	
$\int_{-1}^{1} \lvert x\rvert - x^2 \, dx$	



b) (4 points) Which of the following statements is is true because of the intermediate value theorem? (All functions are differentiable).

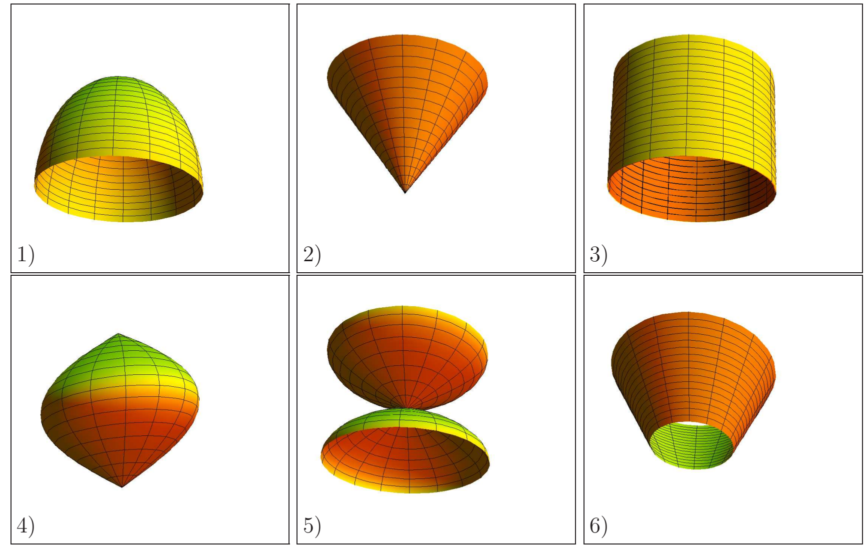
Result	Check
If $f(0) = -1$ and $f(1) = 1$ then there is a root p of f in $(0, 1)$	
If $f(0) = -1$ and $f(1) = 1$ then there is a critical point p of f in $(0, 1)$	
If $f(0) = -1$ and $f(1) = 1$ then there is point where $f(p) = 2$ in $(0, 1)$	
If $f(0) = -1$ and $f(1) = 1$ then there is point where $f'(p) = 2$ in $(0, 1)$	

Problem 3) Matching problem (10 points) No justifications are needed.

a) (6 points) The following integrals match the volumes of solids. Each integral matches exactly one solid.

Integral	Enter 1-6
$\int_0^1 \pi x^2 \, dx$	
$\int_0^1 \pi \, dx$	
$\int_0^1 \pi(1 - x^2) \, dx$	

Integral	Enter 1-6
$\int_0^1 \pi \sin^2(\pi x) \, dx$	
$\int_0^1 \pi(1 + x)^2 \, dx$	
$\int_0^1 \pi \cos^2(\pi x) \, dx$	



b) (4 points) Which of the following results are part of the fundamental theorem of calculus? (any number of results can apply).

Result	Check
$\int_a^b f'(x) \, dx = f(b) - f(a)$	
$\frac{d}{dx} \int_0^x f(t) \, dt = f(x)$	
$\int_0^x f'(t) \, dt = f(x) - f(0)$	
$\int_0^x f(x) \, dx = f'(x)$	

Problem 4) Area computation (10 points)

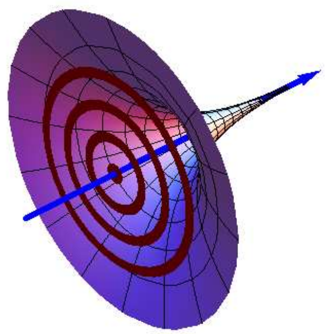
Find the area of the region enclosed by the four curves $x = 2, x = 4, y = 1, y = -2 + \sin(\pi x)$.

Problem 5) Volume computation (10 points)

In this problem we deal with an **extraordinary exponential trumpet**.

a) (7 points) Find the volume of the rotationally symmetric solid for which the radius at position x is $f(x) = \exp(-x)$ and $0 \leq x \leq R$.

b) (3 points) Does the volume limit $R \to \infty$ stay finite? If yes, what is the limit?



Problem 6) Definite integrals (10 points)

Evaluate the following definite integrals

a) (5 points) $\int_1^2 \sqrt{x} + x^2 - \frac{1}{\sqrt{x}} + \frac{1}{x} \, dx$.

b) (5 points) $\int_1^3 \sqrt{1+x} + \frac{4}{1+x^2} \, dx$

Problem 7) Anti derivatives (10 points)

Find the following anti-derivatives

a) (5 points) $\int \frac{1}{\sqrt{1+x}} + \exp(4x) + 2x^5 \, dx$

b) (5 points) $\int \cos(3x) + \sin^2(x) + \frac{1}{\cos^2(x)} \, dx$

Problem 8) Implicit and related rates (10 points)

a) (5 points) The implicit equation $x^4 + y^4 = y^2 + 1$ defines a function $y = y(x)$ near $(x, y) = (1, -1)$. Find the slope $y'(x)$ at this point.

b) (5 points) Take the same equation as before and assume now that $x(t) = t^2$ depends on an external parameter t. This produces a function $y(t)$ near $y = -1$. Find $y'(t)$ at $t = 1$.

Problem 9) Catastrophes (10 points)

Consider the family of functions $f(x) = x^3 + cx$ on the real line.

a) (5 points) Find all critical points of f, depending on c.

b) (2 points) Using the second derivative test, determine which are minima and which are maxima.

b) (3 points) For which value of c does a catastrophe occur?

Problem 10) Applications (10 points)

a) Check that the function

$$f(x) = \begin{cases} 6x(1-x) & 0 \le x \le 1 \\ 0 \; otherwise & \end{cases}$$

is a probability density function.
b) The integral

$$\int_0^1 x f(x) \, dx$$

is called the expectation. Find the expectation.