Lecture 4: Worksheet

Good, the happy, bad and ugly

Jumps, poles and Oscillations are the perils for continuity. We do not have to worry about continuity in most cases, most functions are good. But there are very few mechanisms which bring you in peril. A function can either happily jump, badly rush to infinity or have an ugly oscillation. All cases come from division by zero somewhere.

Good Guys	Bad Guys
$x^2 + 4x + 6$	$1/x$ at 0
$\sin(x), \cos(x)$	$\tan(x)$ at $\pi/2$
$\exp(x)$	$\log|x|$ at 0
$\mathrm{sinc}(x) = \frac{\sin(x)}{x}$	$\sec(x) = \frac{1}{\cos(x)}$ at $\pi/2$

Surprises

$\sin(x)/x$	is continuous at 0
$1/\log|x|$	is continuous at 0
$x\sin(1/x)$	is continuous at 0

Which functions are continuous?

Which of the following functions are continuous?

1. Is $f(x) = \log 1 + |x|$ continuous at $x = 0$?

2. Is $f(x) = \sqrt{|x|}$ continuous at $x = 0$?

3. Is $f(x) = \dfrac{1}{\sqrt{|x|}}$ continuous at $x = 0$?

4. Is $\dfrac{1}{\log|1/x|}$ continuous at $x = 0$?

5. Is $\log(\log|x|)$ continuous everywhere?

6. Is $1/(1 + |x|)$ continuous everywhere?

Enemy of continuity

Oscillations, escape to infinity and jumps are reasons for discontinuity.