

Lecture 6: Worksheet

The exponential function

We illuminate the fundamental theorem for the exponential function $\exp(x) = (1 + h)^{x/h}$. We look again at the case $h = 1$ in which case $\exp(x) = 2^x$ maps positive integers to positive integers.

$$D \exp(x) = \exp(x) .$$

From the fundamental theorem, we get $SD \exp(x) = S \exp(x) = \exp(x) - \exp(0)$ we see

$$S \exp(x) = \exp(x) - 1 .$$

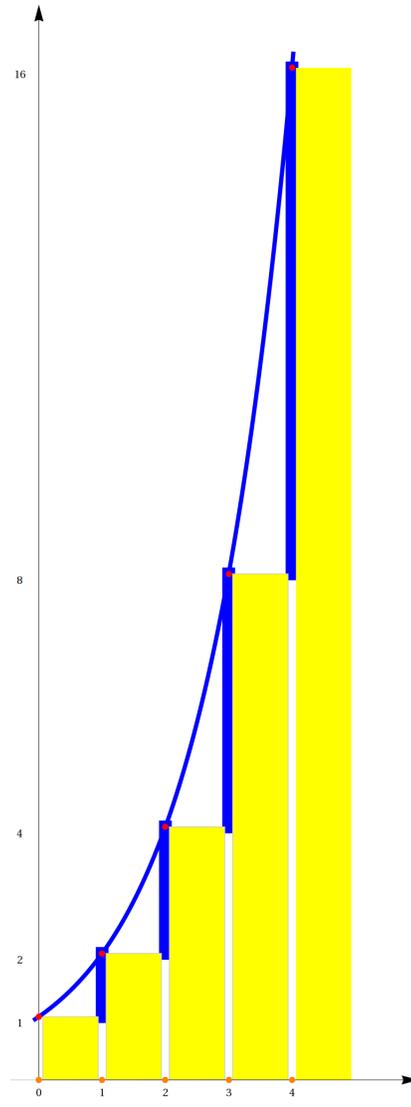
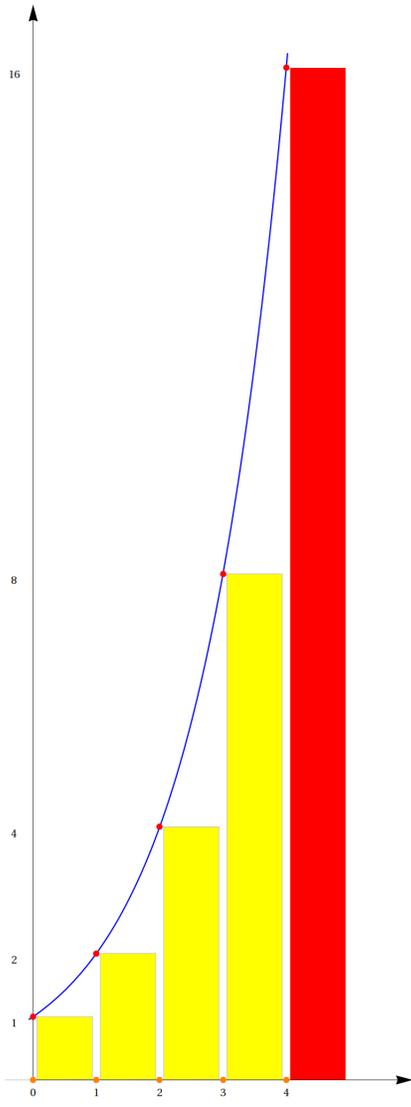
In other words, for the exponential function, we know both the derivative and the integral.

1 The formula $D \exp(x) = \exp(x)$ tells for $x = 3$ that $16 - 8 = 8$.

2 The formula $S \exp(x) = \exp(x) - 1$ tells for $x = 4$ that $1 + 2 + 4 + 8 = 16 - 1$.

3 The identity $DS \exp(x) = \exp(x)$ is illustrated in the left picture for $x = 4$.

4 The identity $SD \exp(x) = \exp(x) - 1$ is illustrated to the right for $x = 4$.



Here are some worked out examples, similar to what we expect you to do for the homework of lecture 6.

The homework should be straightforward. When finding the value of $Sf(x)$, we want to add a constant such that $Sf(0) = 0$. In general, you will not need to evaluate functions and can leave terms like $\sin(5 \cdot x)$ as they are. If you have seen calculus already, then you could do this exercise by writing

$$\frac{d}{dx} f(x)$$

instead of $Df(x)$ and by writing

$$\int_0^x f(x) dx$$

instead of $Sf(x)$. Since we did not introduce the derivative df/dx nor the integral \int_0^x yet, for now, just use the differentiation and integrations rules in the box to the right to solve the problems.

1 Problem: Find the derivative $Df(x)$ of the function $f(x) = \sin(5 \cdot x) + x^7 + 3$.

Answer: From the differentiation rules, we know $Df(x) = 5 \cos(5 \cdot x) + 7x^6$.

2 Problem: Find the derivative $Df(0)$ of the same function $f(x) = \sin(5 \cdot x) + 5x^7 + 3$.

Answer: We know $Df(x) = 5 \cos(5 \cdot x) + 35x^6$. Plugging in $x = 0$ gives 5 .

3 Problem: Find the integral $Sf(x)$ of the function $f(x) = \sin(5 \cdot x) + 5x^7 + 3$.

Answer: From the integration rules, we know $Sf(x) = -\cos(5 \cdot x)/5 + 5x^8/8 + 3x$.

4 Problem: Find the integral $Sf(1)$ of the function $f(x) = \exp(4 \cdot x)$.

Answer: From the integration rules, we know $Sf(x) = \exp(4 \cdot x)/4 - 1/4$. We have added a constant such that $Sf(0) = 0$. Plugging in $x = 1$ gives $\exp(4)/4 - 1/4$.