

Lecture 36: Worksheet

This worksheet as well as the solutions was generated by Sofia, a bot written in the academic year 2003/2004 using grant from the Harvard Provost together with Harvard students **Johnny Carlsson**, **Andrew Chi** and **Mark Lezama**. At that time, people have laughed at the chat bot idea. Now it is big business: Google, Siri, Cortana, Wolfram alpha: these are all AI bots which constantly become more and more sophisticated.

1 Differentiate the following functions:

- a) $f(x) = -3x$
- b) $f(x) = 2 \cos(x)$
- c) $f(x) = 0$

2 Integrate the following functions:

- a) $f(x) = 2(\log(x) + 1)$
- b) $f(x) = 3(\sec^2(x) + 1)$
- c) $f(x) = 2 \cos(x)$

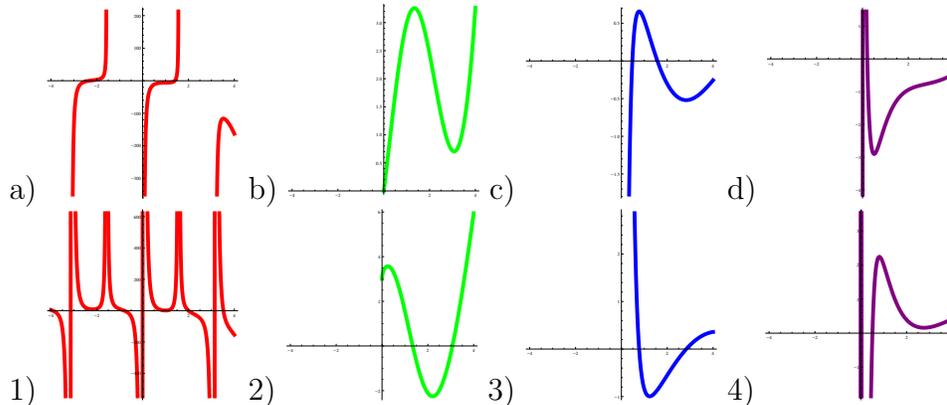
3 Differentiate the following functions:

- a) $f(x) = 4e^{-x}x \cot(x)$
- b) $f(x) = 3(\tan(x) + 3)$
- c) $f(x) = 4x^4 \cos(x)$

4 Integrate the following functions:

- a) $f(x) = 4 \tan^2(x)$
- b) $f(x) = \frac{3}{2\sqrt{x}} - 3 \sec^2(x) + 3$
- c) $f(x) = \frac{x+3 \log(x)+3}{(x+3)^2}$

5 Match the following functions with derivatives:



6 Find the critical points of the following functions:

- a) $f(x) = (x - 9)^2(x - 4)$
- b) $f(x) = (x - 9)(x - 6)$
- c) $f(x) = (x - 5)^2(x - 4)$

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1 Differentiate the following functions:

a) $f(x) = -3x$

b) $f(x) = 2 \cos(x)$

c) $f(x) = 0$

Solution:

a) $f'(x) = -3$

b) $f'(x) = -2 \sin(x)$

c) $f'(x) = 0$

2 Integrate the following functions:

a) $f(x) = 2(\log(x) + 1)$

b) $f(x) = 3(\sec^2(x) + 1)$

c) $f(x) = 2 \cos(x)$

Solution:

a) $\int f(x) = 2x \log(x) + C$

b) $\int f(x) = 3(x + \tan(x)) + C$

c) $\int f(x) = 2 \sin(x) + C$

3 Differentiate the following functions:

a) $f(x) = 4e^{-x}x \cot(x)$

b) $f(x) = 3(\tan(x) + 3)$

c) $f(x) = 4x^4 \cos(x)$

Solution:

a) $f'(x) = -4e^{-x}((x-1)\cot(x) + x\csc^2(x))$

b) $f'(x) = 3\sec^2(x)$

c) $f'(x) = -4x^3(x\sin(x) - 4\cos(x))$

4 Integrate the following functions:

a) $f(x) = 4 \tan^2(x)$

b) $f(x) = \frac{3}{2\sqrt{x}} - 3 \sec^2(x) + 3$

c) $f(x) = \frac{x+3 \log(x)+3}{(x+3)^2}$

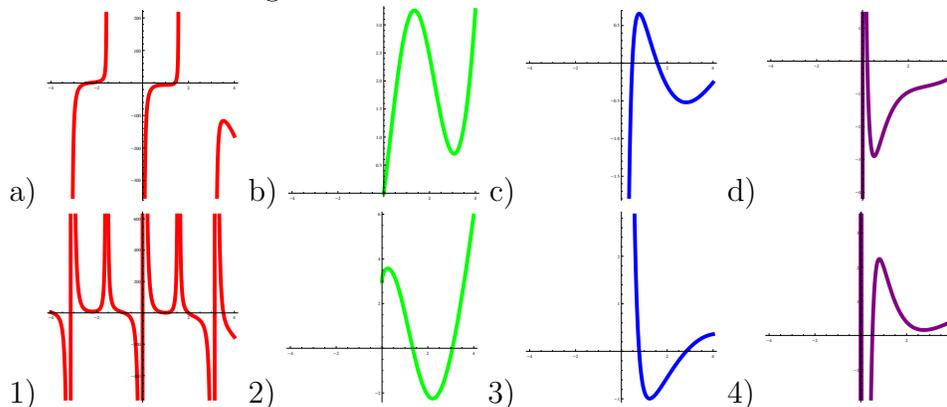
Solution:

a) $\int f(x) = 4(-x + \tan(x) + 2) + C$

b) $\int f(x) = 3(x + \sqrt{x} - \tan(x)) + C$

c) $\int f(x) = \frac{x \log(x)}{x+3} + C$

5 Match the following functions with derivatives:



Solution:

a \rightarrow 1, b \rightarrow 2, c \rightarrow 3, d \rightarrow 4

6 Find the critical points of the following functions:

a) $f(x) = (x - 9)^2(x - 4)$

b) $f(x) = (x - 9)(x - 6)$

c) $f(x) = (x - 5)^2(x - 4)$

Solution:

a) $f'(x) = \left\{ \left\{ x \rightarrow \frac{17}{3} \right\}, \left\{ x \rightarrow 9 \right\} \right\}$

b) $f'(x) = \left\{ \left\{ x \rightarrow \frac{15}{2} \right\} \right\}$

c) $f'(x) = \left\{ \left\{ x \rightarrow \frac{13}{3} \right\}, \left\{ x \rightarrow 5 \right\} \right\}$