4/8/2014: Second midterm exam

Your Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- Except for multiple choice problems, give computations.
- No notes, books, calculators, computers, or other electronic aids are allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) TF questions (20 points) No justifications are needed.

1) T F For any continuous function f we have $\int_0^1 3f(t)\,dt = 3\int_0^1 f(t)\,dt$.

Solution:
Yes this is linearity.

2) T F For any continuous function $\int_0^3 f(t)\,dt = 3\int_0^1 f(t)\,dt$.

Solution:
Looks good but is nonsense.

3) T F For any continuous function $\int_0^1 1 - f(t)\,dt = 1 - (\int_0^1 f(t)\,dt)$.

Solution:
Because the integral over 1 can be computed directly.

4) T F The anti-derivative of $\tan(x)$ is $-\log(\cos(x)) + C$.

Solution:
Differentiate the right hand side to check.

5) T F The fundamental theorem of calculus implies that $\int_1^3 f'(x)\,dx = f(3) - f(1)$.

Solution:
Yes, this is it.

6) T F The integral $\pi\int_0^1 x^2\,dx$ gives the volume of a cone of height 1.

Solution:
Yes the area of a slice is $x^2\pi$.

7) T F The anti-derivative of $1/\cos^2(x)$ is $\tan(x)$.

Solution:
Check.

8) T F

The function $F(x) = \int_0^x \tan(t^2)\, dt$ has the derivative $\tan(x^2)$.

Solution:
The first derivative of F is f.

9) T F

If the area $A(r(t))$ of a disk changes in a constant rate, then the radius $r(t)$ changes in a constant rate.

Solution:
This is a related rates problem.

10) T F

The identity $\frac{d}{dx} \int_1^2 \log(x)\, dx = \log(2) - \log(1)$ holds.

Solution:
We differentiate a constant.

11) T F

If $xy = 3$ and $x'(t) = 1$ at $(3, 1)$ then $y' = 1$.

Solution:
This is a simple example of implicit derivatives.

12) T F

If $f < 1$, then $\int_0^2 f(x)\, dx$ can be bigger than 1.

Solution:
Take $f(x) = 0.6$ for example.

13) T F

An improper integral is an improperly defined definite indefinite integral.

Solution:
If you marked this true, you must have been properly drunk or behaved improperly.

14) T F The anti derivative $F(x)$ of $f(x)$ satisfies $F'(x) = f(x)$.

Solution:
This is the fundamental theorem of calculus

15) T F A parameter value c for which the number of minima are different for parameters smaller or larger than c is called a catastrophe.

Solution:
This is a definition.

16) T F If f is unbounded at 0, then $\int_0^1 f(x)\, dx$ is infinite.

Solution:
The function $\sqrt{x}$ was a counter example.

17) T F If $f(-1) = 0$ and $f(1) = 1$ then $f' = 2$ somewhere on $(-1, 1)$.

Solution:
This is close to the intermediate value theorem.

18) T F The anti-derivative of $\log(x)$ is $x\log(x) - x + C$, where log is the natural log.

Solution:
You might not have known this by heart, but you can check it!

19) T F The sum $\frac{1}{n}[(\frac{0}{n})^2 + (\frac{1}{n})^2 + ... + (\frac{n-1}{n})^2]$ converges to $1/3$ in the limit $n \to \infty$.

Solution:
It is a Riemann sum.

20) T F The **improper integral** $\int_1^\infty \frac{1}{x^2}\, dx$ represents a finite area.

Solution:
We have no problem at infinity.