4/8/2014: Second midterm exam

Your Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- Except for multiple choice problems, give computations.
- No notes, books, calculators, computers, or other electronic aids are allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

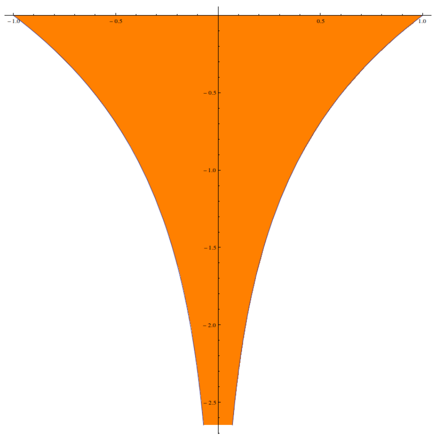
Problem 1) TF questions (20 points) No justifications are needed.

1)	T	F	For any continuous function f we have $\int_0^1 3f(t)\,dt = 3\int_0^1 f(t)\,dt$.
2)	T	F	For any continuous function $\int_0^3 f(t)\,dt = 3\int_0^1 f(t)\,dt$.
3)	T	F	For any continuous function $\int_0^1 1 - f(t)\,dt = 1 - (\int_0^1 f(t)\,dt)$.
4)	T	F	The anti-derivative of $\tan(x)$ is $-\log(\cos(x)) + C$.
5)	T	F	The fundamental theorem of calculus implies that $\int_1^3 f'(x)\,dx = f(3) - f(1)$.
6)	T	F	The integral $\pi\int_0^1 x^2\,dx$ gives the volume of a cone of height 1.
7)	T	F	The anti-derivative of $1/\cos^2(x)$ is $\tan(x)$.
8)	T	F	The function $F(x) = \int_0^x \tan(t^2)\,dt$ has the derivative $\tan(x^2)$.
9)	T	F	If the area $A(r(t))$ of a disk changes in a constant rate, then the radius $r(t)$ changes in a constant rate.
10)	T	F	The identity $\frac{d}{dx}\int_1^2 \log(x)\,dx = \log(2) - \log(1)$ holds.
11)	T	F	If $xy = 3$ and $x'(t) = 1$ at $(3, 1)$ then $y' = 1$.
12)	T	F	If $f < 1$, then $\int_0^2 f(x)\,dx$ can be bigger than 1.
13)	T	F	An improper integral is an improperly defined definite indefinite integral.
14)	T	F	The anti derivative $F(x)$ of $f(x)$ satisfies $F'(x) = f(x)$.
15)	T	F	A parameter value c for which the number of minima are different for parameters smaller or larger than c is called a catastrophe.
16)	T	F	If f is unbounded at 0, then $\int_0^1 f(x)\,dx$ is infinite.
17)	T	F	If $f(-1) = 0$ and $f(1) = 1$ then $f' = 2$ somewhere on $(-1, 1)$.
18)	T	F	The anti-derivative of $\log(x)$ is $x\log(x) - x + C$, where log is the natural log.
19)	T	F	The sum $\frac{1}{n}[(\frac{0}{n})^2 + (\frac{1}{n})^2 + ... + (\frac{n-1}{n})^2]$ converges to $1/3$ in the limit $n \to \infty$.
20)	T	F	The **improper integral** $\int_1^\infty \frac{1}{x^2}\,dx$ represents a finite area.

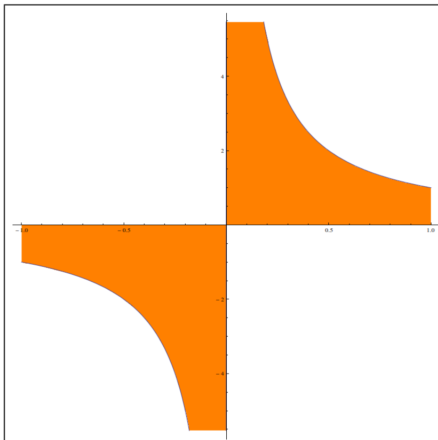
Problem 2) Matching problem (10 points) No justifications are needed.

a) (4 points) Match the following integrals with the regions and indicate whether the integral represents a finite area.

Integral	Fill in 1-4	Finite?
$\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} \; dx$		
$\int_{-1}^{1} \frac{1}{x} \; dx$		
$\int_{-1}^{1} \frac{1}{1+x^2} \; dx$		
$\int_{-1}^{1} \log\|x\| \; dx$		



1)



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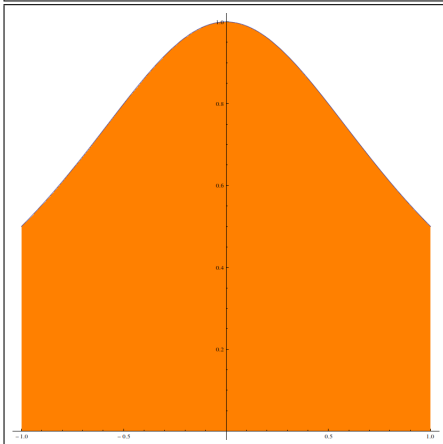

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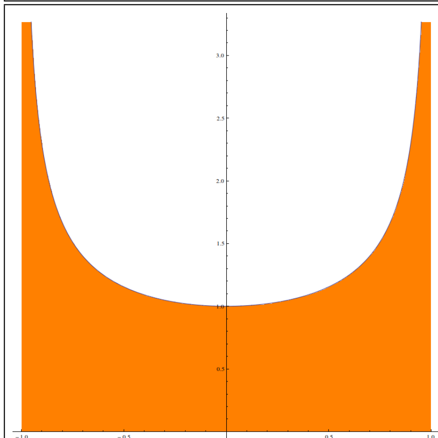


4)

b) (6 points) Which of the following properties are always true. This means which are true for all choices of continuous functions and all choices of a, b, c.

Identity	Check if true
$\int_a^b f(x) \; dx + \int_b^c f(x) \; dx = \int_a^c f(x) \; dx$	
$\int_a^b f(x) \; dx + \int_a^b g(x) \; dx = \int_a^b f(x) + g(x) \; dx$	
$\int_a^b cf(x) \; dx = c \int_a^b f(x) \; dx$	
$\int_a^b f(x)^2 \; dx = (\int_a^b f(x) \; dx)^2$	
$\int_a^a f(x) \; dx = 0$	
$\int_a^b f(x) \; dx = \int_b^a f(x) \; dx$	

Problem 3) (10 points)

Fill in the missing part into the empty box to make a true statement:

a) (2 points)

$\frac{d}{dx} \int_0^x f(t)\, dt = \boxed{}$ by the **fundamental theorem of calculus**.

b) (2 points)

$\int_0^x f(t)\, dt = \boxed{}$ by the **fundamental theorem of calculus**.

c) (2 points)

The **mean value theorem** tells there is exists $a < x < b$ with $\frac{f(b)-f(a)}{b-a} = \boxed{}$

d) (2 points)

A **probability distribution** satisfies $\int_{-\infty}^{\infty} f(x)\, dx = 1$ and $\boxed{}$ for all x.

e) (2 points)

For an improper integral $\int_a^b f(x)\, dx$, either $a = \infty$ or $b = \infty$ or f is $\boxed{}$ on $[a, b]$.

Problem 4) Area computation (10 points)

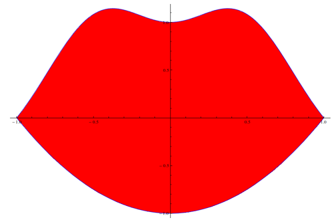
The region enclosed by the graphs of

$$f(x) = x^2 - 1$$

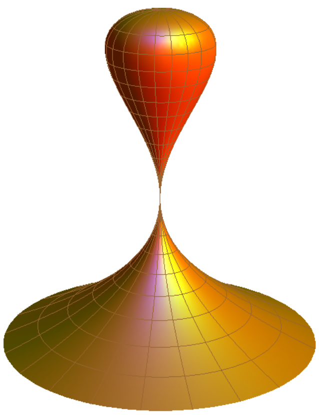
and

$$g(x) = 1 - x^2 + (1 - \cos(2\pi x))/6$$

models of the lips of **Rihanna**. Find the area.



Problem 5) Volume computation (10 points)



The **kiss** is a solid of revolution for which the radius at height z is

$$z^2\sqrt{1-z}$$

and where $-1 \leq z \leq 1$. What is the volume of this solid? The name "kiss" is the official name for this quartic surface. Indeed, the top part has the shape of a **Hershey Kiss**. P.S. Creative **"exam product placement"** like this has been invented and patented by Oliver himself ...

Problem 6) Definite integrals (10 points)

Evaluate the following definite integrals. Each of the problems produces a numerical answer.

a) (2 points) $\int_0^1 \sqrt{1+x}\, dx$.

b) (2 points) $\int_0^1 \frac{1}{1+x^2}\, dx$

c) (2 points) $\int_2^e \frac{5}{3+x}\, dx$

d) (2 points) $\int_0^1 \frac{1}{\sqrt{1-x}}\, dx$.

e) (2 points) $\int_0^1 (x+1)^{10}\, dx$

Problem 7) Anti derivatives (10 points)

Find the following anti-derivatives:

a) (2 points) $\int e^{23x} - x^{23}\, dx$

b) (2 points) $\int \frac{2}{x+3} + x^{1/23}\, dx$

c) (2 points) $\int \frac{23}{1+x} + 23\tan(x)\, dx$

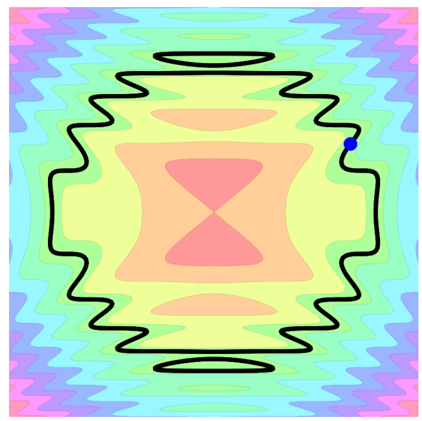
d) (2 points) $\int \frac{1}{\sin^2(x)} + \frac{1}{x^3}\, dx$

e) (2 points) $\int \cos^2(3x)\, dx$

Jim Carrey in the movie "The number 23"

Problem 8) Implicit differentiation and related rates (10 points)

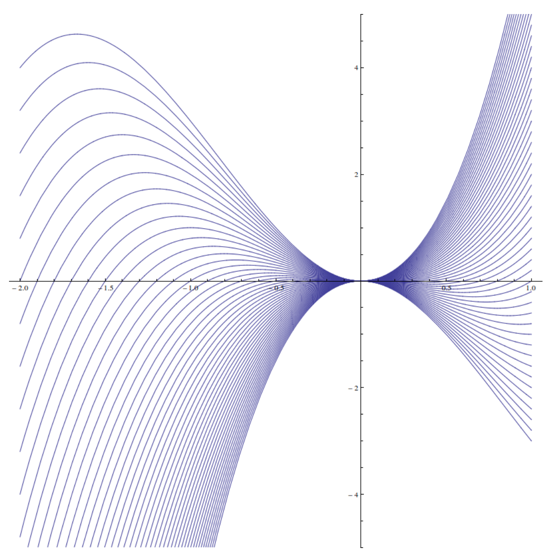
a) (5 points) Find the slope y' of the curve $x^2 + y^2 + \sin(x^2 - 4y^2) = 5$ at $x = 2, y = 1$.



b) (5 points) We deal with functions $x(t), y(t)$ where t is time. If $x^4 + 2y^4 = 3$ and $x'(t) = t$, find $y'(t)$ at the point $(x, y) = (1, 1)$.

Problem 9) Catastrophes (10 points)

We look at the one-parameter family of functions $f_c(x) = 2x^3 + cx^2$, where c is a parameter.



a) (2 points) Find the critical points of $f_3(x)$.

b) (2 points) Find the critical points of $f_{-3}(x)$.

c) (2 points) Check that 0 is always a critical point.

d) (2 points) For which c is 0 a minimum?

e) (2 points) For which c does the catastrophe occur?