4/8/2014: Second midterm practice C

Your Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- Except for multiple choice problems, give computations.
- No notes, books, calculators, computers, or other electronic aids are allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) TF questions (20 points) No justifications are needed.

1) T F The formula $\int_0^x f''(x)\,dx = f'(x) - f'(0)$ holds.

2) T F The area of the upper half disc is the integral $\int_{-1}^1 \sqrt{1-x^2}\,dx$

3) T F If the graph of the function $f(x) = x^2$ is rotated around the interval $[0,1]$ in the x axes we obtain a solid with volume $\int_0^1 \pi x^4\,dx$.

4) T F The function $f(x) = e^x$ is the only anti derivative of e^x.

5) T F If f has a critical point at 1, then $F(x) = \int_0^x f(t)\,dt$ has an inflection point at 1.

6) T F Catastrophes are parameter values c for a family of functions $f_c(x)$, for which a local minimum of f_c disappears.

7) T F The volume of a cylinder of height and radius 1 minus the volume of a cone of height and radius 1 is half the volume of a sphere of radius 1.

8) T F Rolle's theorem tells that if $0 < c < 1$ is a critical point of f and $f(0) = f(1)$, then the critical point is in the interval $[0,1]$.

9) T F Rolle also introduced the notation $|x|^{1/3}$ for roots.

10) T F Integrals are linear: $\int_0^x f(t) + g(t)\,dt = \int_0^x f(t)\,dt + \int_0^x g(t)\,dt$.

11) T F The function $\mathrm{Li}(x) = \int_2^x dt/\log(t)$ has an anti-derivative which is a finite construct of trig functions.

12) T F There is a region enclosed by the graphs of x^5 and x^6 which is finite and positive.

13) T F The integral $\int_{-1}^1 1/x^4\,dx = -1/(5x^5)|_{-1}^1 = -1/5 - 1/5 = -2/5$ is defined and negative.

14) T F Gabriel's trumpet has finite volume but infinite surface area.

15) T F A function $f(x)$ is a probability density function, if $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

16) T F The mean of a function on an interval $[a,b]$ is $\int_a^b f(x)\,dx$.

17) T F The cumulative probability density function is an antideriative of the probability density function.

18) T F The integral $\int_{-\infty}^{\infty}(x^2-1)\,dx$ is finite.

19) T F The total prize is the derivative of the marginal prize.

20) T F The acceleration is the anti-derivative of the velocity.

Problem 2) Matching problem (10 points) No justifications are needed.

Match the following functions with their anti derivatives. Of course only 6 of the 30 functions will appear.

Function	Antiderivative Enter 1-30
$\cos(3x)$	
$\sin(3x)$	
$3x$	

Function	Antiderivative Enter 1-30
$1/(3x)$	
$\tan(3x)$	
$1/(1+9x^2)$	

1) $\sin(3x)$
2) $-\sin(3x)/3$
3) $\sin(3x)/3$
4) $-3\sin(3x)$
5) $3\sin(3x)$
6) $\cos(3x)$
7) $-\cos(3x)/3$
8) $\cos(3x)/3$
9) $-3\cos(3x)$
10) $3\cos(3x)$
11) $\log(x)/3$
12) $1/(3-x)$
13) $1/(3x)$
14) $\log(x/3)$
15) $-1/(3x^2)$
16) $3x^2$
17) $x^2/2$
18) $3x^2/2$
19) 3
20) x^2
21) $\arctan(3x)/3$
22) $3\arctan(3x)$
23) $1/(1+9x^2)$
24) $3/(1+9x^2)$
25) $-3/(1+x^2)$
26) $1/\cos^2(3x)$
27) $\log(\cos(3x))$
28) $-\log(\cos(3x))/3$
29) $\log(\cos(3x))/3$
30) $3/\cos^3(3x)$

Problem 3) Matching problem (10 points) No justifications are needed.

Which of the following formulations is a Riemann sum approximating the integral $\int_0^3 f(x)\,dx$ of $f(x) = x^2$ over the interval $0, 3$.

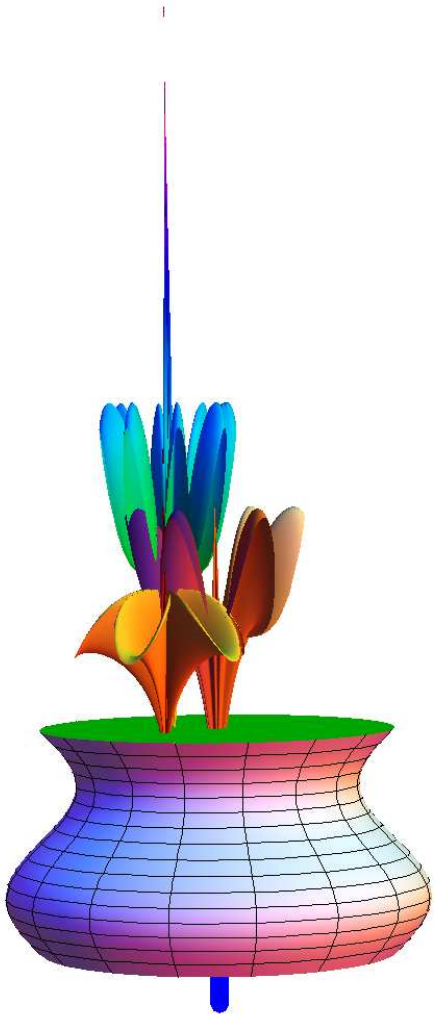
Sum	Check if this is the Riemann sum.
$n\sum_{k=0}^{n-1}(3k/n)^2$	
$\frac{1}{n}\sum_{k=0}^{n-1}(3k/n)^2$	
$n\sum_{k=0}^{3n-1}(k/n)^2$	
$\frac{1}{n}\sum_{k=0}^{3n-1}(k/n)^2$	

Problem 4) Area computation (10 points)

Find the area of the region enclosed by the three curves $y = 6 - x^2, y = -x$ and $y = x$ which is above the x axes.

Problem 5) Volume computation (10 points)

Emma R. grows magical plants in a pot which is a rotationally symmetric solid for which the radius at position x is $5 + \sin(x)$ and $0 \leq x \leq 2\pi$. Find the volume of the pot.



Problem 6) Definite integrals (10 points)

Find the following definite integrals

a) (5 points) $\int_1^2 x^{1/5} + x^4 + 1/x \; dx$.

b) (5 points) $\int_1^3 2x + \sin(x-1) + \cos(x+2) \; dx$

Problem 7) Anti-derivatives (10 points)

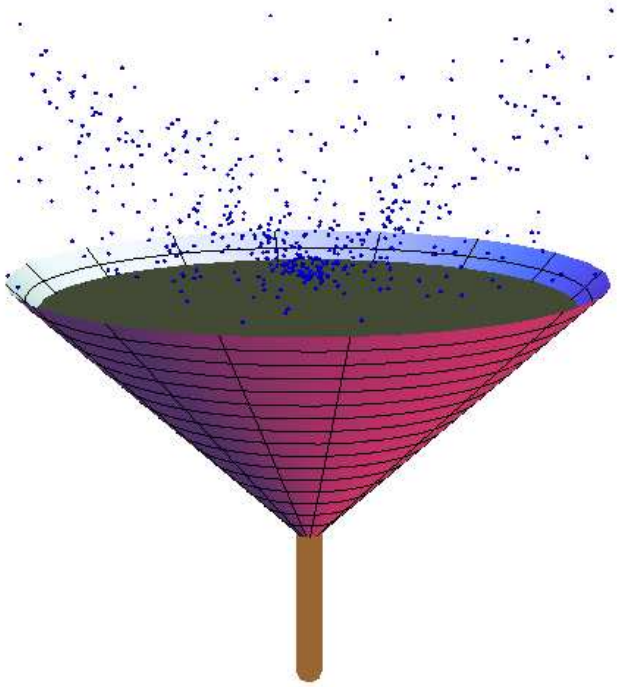
Find the following anti-derivatives

a) (5 points) $\int \frac{3}{\sqrt{1-x^2}} + x^4 + \frac{1}{1+x^2} \; dx$

b) (5 points) $\int \frac{1}{x-2} + \frac{1}{x+4} + \frac{2}{x-1}\, dx$

Problem 8) Related rates (10 points)

A coffee machine has a filter which is a cone of radius z at height z. Coffee drips out at a rate of 1 cubic centimeter per second. How fast does the water level sink at height $z = 10$?



Problem 9) Implicit differentiation (10 points)

Find the derivatives $y' = dy/dx$ of the following implicitly defined functions:

a) (5 points) $x^5 + 3x + y = e^y$.

b) (5 points) $\sin(x^2 - 2y) = y - x$.

Problem 10) Improper integrals (10 points)

Evaluate the following improper integrals or state that they do not exist

a) (3 points) $\int_1^\infty 1/\sqrt{x}\ dx$.

b) (2 points) $\int_0^1 \sqrt{x}\ dx$.

c) (3 points) $\int_0^\infty 2xe^{-x^2}\ dx$.

d) (2 points) $\int_0^\infty \frac{1}{x}\ dx$