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CARTESIAN COORDINATE SYSTEMS. Points on the line are labeled by 1 coordinate x, points in the **plane** are labeled by 2 coordinates (x, y). Points in space are determined by three coordinates (x, y, z).

1D space = line = 2 half lines   2D space = plane = 4 quadrants   3D space = space = 8 octants
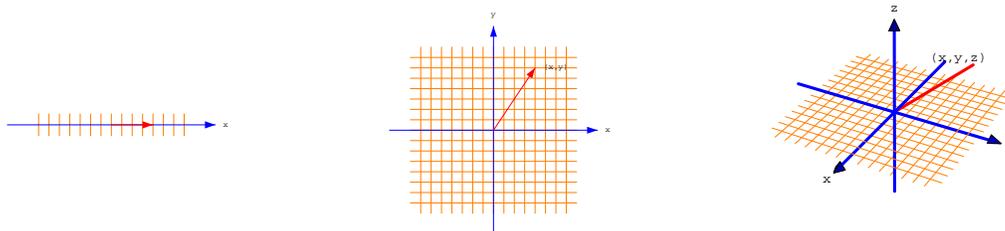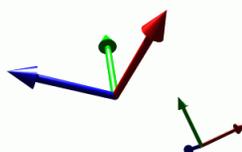

CHOICE OF COORDINATE SYSTEM
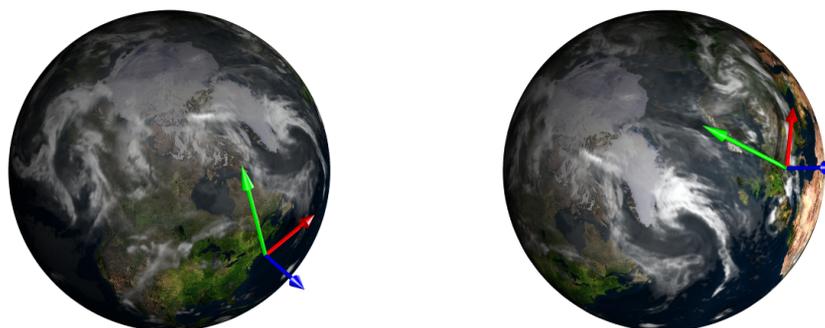
DISTANCE. The distance between two points $P = (x, y, z)$ and $Q = (a, b, c)$ is

$$d(P, Q) = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} \ .$$

This can be derived from Pythagoras (suggested problem).

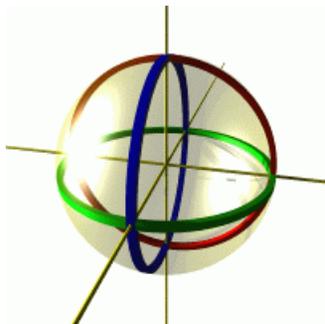PARITY.   Orientation of coordinate system

Use "right hand rule" to distinguish.

GEOMETRICAL OBJECTS. **curves**, **surfaces** and **bodies** are examples of geometrical objects described with **functions of several variables**. We look at them in this section to get some feel about space. The objects will be treated later in more detail.

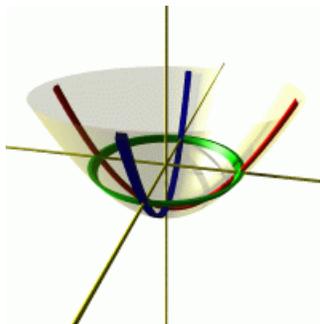$$f(x, y, z) = ax^2 + by^2 + cz^2 + dxy + exz + fyz + gx + hy + kz + m$$

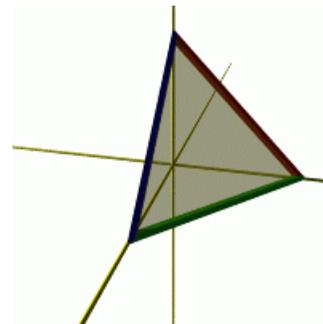the the surface is called a **quadric**.

SPHERE	PARABOLID	PLANE
$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$	$(x-a)^2 + (y-b)^2 - c = z$	$ax + by + cz = d$
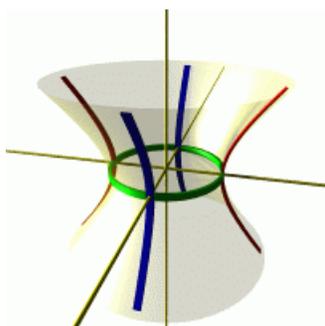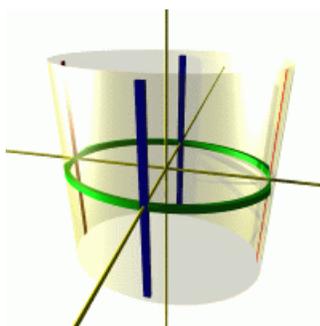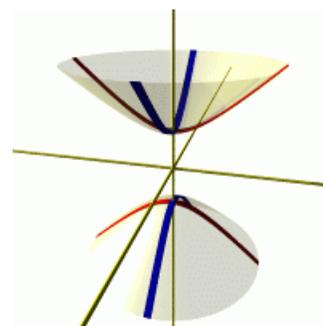
HYPERBOLOID I	CYLINDER	HYPERBOLOID II
$(x-a)^2 + (y-b)^2 - (z-c)^2 = r^2$	$(x-a)^2 + (y-b)^2 = r^2$	$(x-a)^2 + (y-b)^2 - (z-c)^2 = -r^2$

PROBLEM. Given a quadric $f(x, y, z) = 0$. How do we draw it?

- Manipulate the function to identify $f = 0$ as a known surface (**completing the square**).

- Draw **traces**, intersections of the surface with the coordinate planes $x = 0, y = 0, z = 0$. Find **intercepts**, intersections of the surface with the coordinate axes.

- If a variable (i.e. z) does not appear in $f(x, y, z)$, we have a **cylinder** $f(x, y) = 0$. We draw the shape in the (x, y) plane and sweep it out. An example of a cylinder is $x^2 + 2 * y^2 - 1 = 0$.

- Can we solve for one of the variables? For example, if we can transform $f(x, y, z) = 0$ to $z = g(x, y)$, then the surface is a **graph.** An example is the paraboloid $x^2 + y^2 - z = 0$.

EXAMPLE. $f(x, y, z) = x^2 - 2x + y^2 - 4y - z^2 - 5 = 0$ is equivalent to $(x-1)^2 + (y-2)^2 - z^2 = 5 + 1 + 4 = 10$. The xy-trace is the circle $(x-1)^2 + (y-2)^2 = 10$, the xz-trace is the hyperbola $(x-1)^2 - z^2 = 6$, the yz-trace is the hyperbola $(y-2)^2 - z^2 = 9$. The x-intercepts are $0, 3$, the y-intercepts are $y = 0, 1$, the z-intercepts are $\pm\sqrt{5}$. The surface is a **one-sheeted hyperboloid.** We could write the $z \geq 0$ part as a **graph** $z = \sqrt{5 - x^2 - 2x + y^2 - 4y}$ and the $z \leq 0$ part as $z = -\sqrt{5 - x^2 - 2x + y^2 - 4y}$.

SUGGESTED PROBLEMS. Section 1.1 pgs. 11-15. 1,7,9,11,15,21,25.