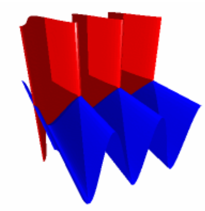
Last lessson: spiral as intersection of surfaces $x = \cos(z), y = \sin(z)$.



Suggested Reading: pgs 35-37 number 1, 3, 5, 7, 9. pg 46 number 1. Also consider:

1. A kite string exerts a 12-lb pull (the force F has magnitude 12) on a kite and makes an angle of 45 degrees with the horizontal. What are the horizontal and vertical components of the force vector?

2. Pull a wagon with a force F of magnitude 10-lbs by a rope which makes an angle of 30 degrees with the horizontal. What are the horizontal and vertical components of the force?

VECTORS. Two points $P_1 = (x_1, y_1, z_1)$, $Q = P_2 = (x_2, y_2, z_2)$ determine a **vector** $v = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$. It points from P_1 to P_2 and we can write $P_1 + v = P_2$.
Points P in space are in one to one correspondence to vectors pointing from 0 to P. The numbers v_i in a vector $v = (v_1, v_2, v_3)$ are also called **coordinates** of the vector.

REMARK: vectors can be drawn **everywhere** in space. If a vector starts at 0, then the vector $v = (v_1, v_2, v_3)$ points to the point (v_1, v_2, v_3). That's is why one can identify points $P = (a, b, c)$ in space with a vector $v = (a, b, c)$. Two vectors which are translates of each other are considered equal. (*[1])

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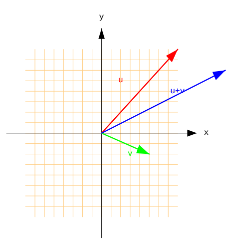
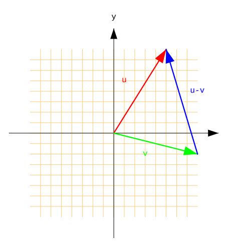
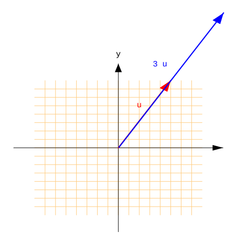
ADDITION SUBTRACTION, SCALAR MULTIPLICATION.



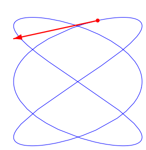




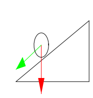
BASIS VECTORS. The vectors $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$ and $\mathbf{k} = (0, 0, 1)$ are called **basis vectors**.

Every vector $v = (v_1, v_2, v_3)$ can be written as a sum of basis vectors: $v = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$.

WHERE DO VECTORS OCCUR?
Velocity: if $(f(t), g(t), h(t))$ is a curve, then $v = (f'(t), g'(t), h'(t))$ is the **velocity vector** at the point $(f(t), g(t), h(t))$.

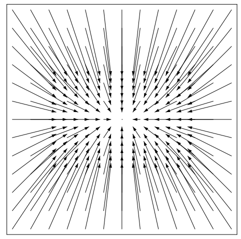


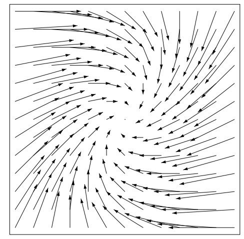
Forces: static problems involve the determination of a force on objects.



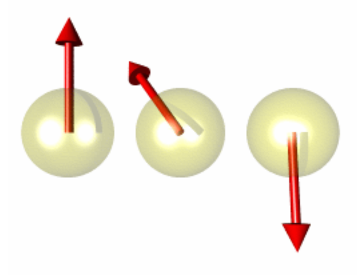
[1] The remark is a common point of confusion. Mathematicians call vectors **affine vectors** and restrict the word **vectors** to affine vectors attached to zero. Calculus courses don't want to add too much terminology and call affine vectors simply **vectors**. Sometimes, vectors attached to 0 are called **bound vectors**. Most courses (like this one) opt for more simplicity and use only the word **vectors** - considering the resulting confusion not grave enough to worry about.

Fields: fields like electromagnetic or gravitational fields or velocity fields in fluids are described with vectors.

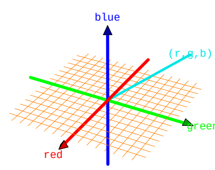


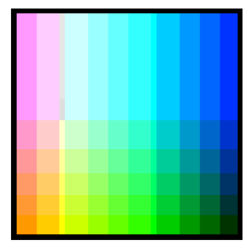


Qbits: in quantum computation, one does not work with bits, but with qbits, which are vectors.



Color Any color can be written as a vector $v = (r, g, b)$, where r is the red component, g is the green component and b is the blue component.





VECTOR OPERATIONS: The addition and scalar multiplication of vectors satisfies some properties. They are all "obvious" (there is no point in memorizing them).

$u + v = v + u$	commutativity
$u + (v + w) = (u + v) + w$	additive associativity
$u + 0 = u + 0$	null vector
$r * (s * v) = (r * s) * v$	scalar associativity
$(r + s)v = v(r + s)$	distributivity in scalar
$r(v + w) = rv + rw$	distributivity in vector
$1 * v = v$	one element

LENGTH OF A VECTOR.
The length $||v||$ of a vector v is the distance from the beginning to the end of the vector.
EXAMPLE. The length of the vector $v = (3, 4, 5)$ is $||v|| = \sqrt{50} = 5\sqrt{2}$.

TRIANGLE INEQUALITY: $||u + v|| \leq ||u|| + ||v||$.

UNIT VECTOR.
A vector of length 1 is called a **unit vector**. If v is a vector which is not zero, then $v/||v||$ is a unit vector.
EXAMPLE: If $v = (3, 4)$, then $v = (2/5, 3/5)$ is a unit vector in the plane.

PARALLEL VECTORS.
Two vectors v and w are called **parallel**, if $v = rw$ with some constant w.