Suggested Problems: pg 46 number 3, pgs 55-58 number 5, 7, 9, 17a-c,e, 19, 21. Also do: pgs 55-58 number 8, 12.

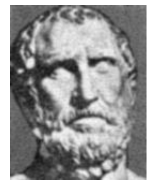
DERIVATIVES. if $r(t) = (x(t), y(t), z(t))$ is a vector valued function describing a curve, then

$$r'(t) = (x'(t), y'(t), z'(t)) = (\dot{x}, \dot{y}, \dot{z})$$

is called the **derivative** of r. (The dot notation is very common, when the parameter is time.) The derivative is also called the **velocity**. The length of the velocity vector is called the **speed**. The derivative of the velocity is called **acceleration**. While the velocity vector is tangent to the curve, the acceleration can point in any direction.

EXAMPLE. If $r(t) = (\cos(3t), \sin(2t), 2\sin(t))$, then $r'(t) = (-3\sin(3t), 2\cos(2t), 2\cos(t))$.

WHAT IS MOTION? **The paradoxon of Zeno of Elea**: "If we look at a body at a specific time, then the body is fixed. Having it fixed at each time, there is no motion".



WHAT IS A DERIVATIVE? The derivative or rate of change is a **limit**. It can be approximated by the vector $(r(t+dt) - r(t))/dt$ where dt is a small number. If dt goes to zero, and the limit exists the velocity exists at this point. If $r(t) = P + vt$ is a line, then $r'(t) = v$.

EXAMPLES OF VELOCITIES.

Electrons in Metals:	0.005 m/s
Person walking:	1.5 m/s
Car:	15-50 m/s
Signals in nerves:	40 m/s
Aeroplane:	70-900 m/s
Sound in air:	Mach1=340 m/s
Satellite:	1200 m/s
Speed of bullet:	1200-1500 m/s
Earth around the sun:	30'000 m/s
Sun around galaxy center:	200'000 m/s
Light in vacuum:	300'000'000 m/s

EXAMPLES OF ACCELERATIONS.

Train:	0.1-0.3 m/s^2
Car:	3-8 m/s^2
Combat plane (F16) (blackout):	9G=90 m/s^2
Ejection from F16:	14G=140 m/s^2.
Free fall:	1G = 9.81 m/s^2
Electron in vaccum tube:	10^{15} m/s^2

INTEGRATION. If $v(t) = (x(t), y(t), z(t))$ is a curve, then $\int_0^t v(t)\ dt$ is defined as $(\int_0^t x(t)\ dt, \int_0^t y(t)\ dt, \int_0^t z(t)\ dt)$.

APPLICATION. A flight recorder in a space object records the accelerations $(a(t), b(t), c(t) = (\sin(2t), \sin(t), t)$ in x, y, z direction. The accelerations are accessible because they are proportional to forces, the device can measure. If the plane is at rest at $(0,0,0)$ when $t = 0$, where is it at $t = 10\pi$?

ANSWER. We know $r''(t) = (\cos(t), \sin(t), t)$. By integration, we obtain $\int_0^t r''(t)\ dt = r'(t) = (\cos(t)/2, -\cos(t), t^2/2)$ and $r(t) = (-\sin(2t)/4, -\sin(t), t^3/6)$. At $t = 10\pi$, we have $r(10\pi) = (0, 0, 1000\pi^3/6)$.

ARC LENGTH. If $r(t)$ is a curve defined on some parameter interval $[a, b]$, and $v(t) = r'(t)$ is the velocity and $||v(t)||$ is the speed, then $\int_a^b ||v(t)||\ dt$ is called the **length of the curve**.

Written out, the formula is

$$L = \int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\ dt\ .$$

$10\pi\sqrt{2}$.

REMARK. Often we will not be able to find a closed formula for the lenght of a curve. For example the Lissajoux figure $r(t) = (\cos(3t), \sin(5t))$ has length $\int_0^{2\pi} \sqrt{9\sin^2(3t) + 25\cos^2(5t)}\, dt$ which can be evaluated only numerically.

NEWTONS LAW. Newton second law says
$m\ddot{t} = mr''(t) = F(t)$
where $F(t)$ is the external force acting on the body and m is the **mass** of the body.

GRAVITY. If $F(t) = (0, 0, -gm)$, then a body feels a constant acceleration towards the ground: $\ddot{r} = -g = -9.81m/s^2$.

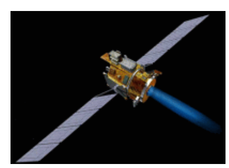
QUESTION. If we drop a body from height h, how long does it take to hit the ground?
ANSWER. The position at time t is $(0, 0, h - gt^2/2)$. For $t = \sqrt{2h/g}$, we are at the ground. For example, if $h = 10$ meters, then we have wait about 1.4 seconds.

PROBLEM. In the movie "Six days, seven nights" (with Harrison Ford), pirats shoot with a cannon onto the plane of the heros. They aim however vertically up onto the plane. The 25mm bullet has an initial speed of 35m/s. How much time do the pirats have until the boat is hit by their own bullet? Assume $g = 10$.
ANSWER. If $v(t)$ is the speed of the cannon shell, then $v'(t) = -g$ and $v(t) = 35 - 10 * t$. The velocity is zero after 3.5 seconds. The pirats have therefore 7 seconds to leave the boat.

DEEP SPACE. This weekend on Saturday the 22. September 2001, the ion drive rocket engine powered **deep space** craft made a risky flyby at **Borelly**, a comet which encircles the sun in about 7 years. The space craft passed the comet at a distance of 2'200km with a speed of 61'000 km/h. Such high speed flybies are tricky and risky. (In July 28, Deep space passed the asteriod Braille and due to an error in the autonavigation system, the camera was unable to lock onto the asteriod.)

QUESTION: How fast did one have the turn the camera of **deep space** at the moment of the closest encounter with the comet?

ANSWER: If the probe would circle around the comet in a distance of $r = 2'200km$ at a speed of 61'000 km/h. It makes $61'000/(2\pi 2'200) = 4.4$ rounds per hour. This means about 1/2 degree in a second.



HISTORY OF NEWTON'S LAW.
Ancient Greek philosophers thought that the motions of the stars and planets were unrelated to events on the earth. The understanding of gravity changed with Galileo, Kepler, Brahe and Newton in the 16'th century. Galileo realized that the gravitational acceleration is independent of the mass of the body. By 1666 Newton did not understand the mechanics of circular motion yet. In 1666 he imagined that the Earth's gravity is influenced the Moon, counterbalancing its centrifugal force. From his law of centrifugal force and Kepler's third law of planetary motion, Newton deduced the inverse-square law. In 1679 Newton corresponded with Hooke who had written to Newton claiming "... that the attraction always is in a duplicate proportion to the distance from the center reciprocall". But Newton then himself derived Kepler's laws from the law of central forces. (See Book: "Huygens and Barrow, Newton and Hooke" by V.I. Arnold)