Homework:

- pgs 263-264 number 2,6,8

- pgs 124-127 number 2,4,14,18

- Let $F = (2xy^2 + 3x^2, 2yx^2)$. Compute the line integral of F from $(0,0)$ to $(1,1)$ along the following curves:
  a) The diagonal x=y
  b) Along the x axis from $x = 0$ to $x = 1$, then from $(1,0)$ to $(1,1)$ along the line $x = 1$.
  c) Along the y axis from $y = 0$ to $y = 1$, then from $(0,1)$ to $(1,1)$ along the line $y = 1$.
  d) Exhibit a potential function for F and use the fundamental theorem for line integrals.

- For each of the following, find values for the constants a,b which make the given vector field conservative:
  a) $F = (ax^3y + by^2, x^4 + yx)$
  b) $F = (\sin(y) + by\cos(x), ax\cos(y) + \sin(x))$
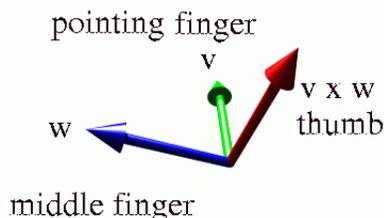 c) $F = (aye^{xy} + y^2, -xe^{xy} + 2yx^b)$
  d) $F = (3x^ay^b, 4x^3y^3)$

VECTORS.
coordinates $v = (v_1, v_2, v_3)$.
addition $v + w = (v_1 + w_1, v_2 + w_2, v_3 + w_3)$.
scaling $\lambda v = (\lambda v_1, \lambda v_2, \lambda v_3)$.
dot product $(v_1, v_2, v_3) \cdot (w_1, w_2, w_3) = v_1w_1 + v_2w_2 + v_3w_3$ is a scalar. The angle between v and w satisfies: $v \cdot w = |v||w|\cos(\phi)$.
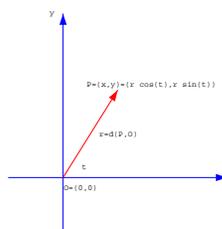

pointing finger
v
v x w
thumb
w
middle finger

cross product $v \times w = (v_2w_3 - v_3w_2, v_3w_1 - v_1w_3, v_1w_2 - v_2w_1)$ is a vector orthogonal to both v and w with length $|v||w|\sin(\phi)$. Possible use: if P, Q, R points on a plane $\Rightarrow (P - Q) \times (R - Q)$ orthogonal to plane.

triple product $u \cdot (v \times w)$ is volume of parallel-epiped spanned by u, v, w.

polar coordinates:

$$(r, \phi) = (\sqrt{x^2 + y^2}, \arctan(y/x))$$
$$(x, y) = (r\cos(\phi), r\sin(\phi))$$
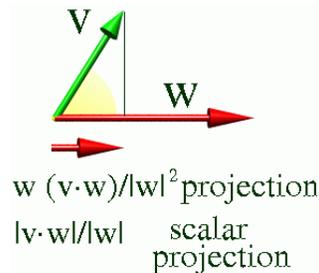

GEOMETRY.
orthogonality $v \cdot w = 0$. Examples: v is orthogonal to $v \times w$. $n = (a, b, c)$ is orthogonal to the plane $ax + by + cz = d$. The vectors $(a, b) = (-b, a)$ are orthogonal in the plane.
parallel $v = \lambda w$, Q, R points on line $X(t) = P + tv \Rightarrow Q - R$ is parallel to v.
angle between two vectors v, w satisfies $\cos(\alpha) = v \cdot w / (|v||w|)$.
projection $w(v \cdot w)/|w|^2$
scalar projection $|(v \cdot w)|/|w|$

V
W
w (v·w)/|w|² projection
|v·w|/|w| scalar projection

distance Point-Point (3D) $d(P, Q) = |P - Q|$,
distance Point-Plane (3D) $d(P, n \cdot x = d) = |(P - Q) \cdot n|/|n|$
distance Point-Line (3D) $d(P, Q + tv) = |(P - Q) \times v|/|v|$.
distance Line-Line (3D) $d(P + tu, Q + tv) = |(P - Q) \cdot (v \times w)|/|v \times w|$.

distance Point-Point (2D) $d(P, Q) = |P - Q|$,
distance Point-LINE (2D) $d(P, n \cdot x = d) = |(P - Q) \cdot n|/|n|$
distance Point-Line (2D) $d(P, Q + vt) = |(P - Q) \times v|/|v|$

velocity $r'(t) = X'(t) = (x'(t), y'(t), z'(t))$. The velocity vector is **tangent to the curve**.

speed $|X'(t)| = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}$. The speed is a **scalar**.

lines curve $r(t) = P + tv$ or $(x-a)/A = (y-b)/B = (z-c)/C$ (intersection of two planes). P is on the curve, v is parallel to the line, $(a,b,c), (a+A, b+B, c+C)$ are on the curve, can take $v = (A, B, C)$.

level curves curves of the form $f(x,y) = c$: Examples: **ellipse** $4x^2 + 9y^2 = 1$, **line** $ax + by = c$. Remember: $\nabla f(x,y)$ is orthogonal to the curve at (x, y).

polar curves $(r(t), \phi(t))$ is $(x(t), y(t)) = (r(t)\cos(\phi(t)), r(t)\sin(\phi(t)))$ in Cartesian coordinates. Examples: the rose $r(t) = \cos(10t)$, $\phi(t) = t$ is the curve $(x(t), y(t)) = (\cos(10t)\cos(t), \cos(10t)\sin(t))$.

length of curve $\int_a^b |r'(t)|\, dt = \int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\, dt$.
curve-plane intersection. $r(t) = (x(t), y(t), z(t)) = P + tv = (P_+ tv_1, b + tv_2, c + tv_3)$ intersects $ax + by + cz = d$: solve $a(P_1 + tv_1) + b(P_2 + tv_2) + c(P_3 + tv_3) = d$ for t.

SURFACES.
surfaces $F(x, y, z) = c$ (level surfaces).

normal vector $\nabla F(x, y, z)$ is normal vector to surface $F(x, y, z) = c$

graphs $z = f(x, y)$ are special surfaces: $F(x, y, z) = f(x, y) - z = 0$.

level surfaces $F(x, y, z) = c$ are also called **contour surfaces**.

plane $ax + by + cz = d$, remember, (a, b, c) is normal to the plane.

sphere $(x-a)^2 + (y-b)^2 + (z-c)^2 = r$, Example: $x^2 + y^2 + z^2 = 1$ **unit sphere**.

one sheeted hyperboloid $x^2 + y^2 - z^2 = r^2$.

two sheeted hyperboloid $x^2 + y^2 - z^2 = -r^2$.

ellipsoid $a^2 x^2 + b^2 y^2 + c^2 z^2 = r^2$

paraboloid $x^2 + y^2 - z = d$.

plane-plane intersection=angle between normal vectors. Example: angle between $x^2 + y^2 + z^2 = 32$ and $x^2 + y^2 - z^2 = 18$ at $(5, 0, \sqrt{7})$ is angle between $(10, 0, 2\sqrt{7})$ and $(10, 0, -2\sqrt{7})$ which is $\arccos(9/16)$.

trace is the intersection between coordinate plane and surface.

intercept is the intersection between a coordinate axes and surface.

INTEGRATION/DIFFERENTIATION
velocity If $r(t) = (x(t), y(t), z(t)$, then $r'(t) = (x'(t), y'(t), z'(t))$.

gradient $\nabla F(x, y, z) = (F_x, F_y, F_z)$ (not subject of this midterm)

integration $\int r(t)dt = (\int x(t)dt, \int y(t)dt, \int z(t)dt)$.

line integrals (not subject of this midterm) $\int F(r(t)) \cdot r'(t)\, dt$ is scalar while $\int r(t)\, dt$ is a vector.

product rule for dot product $\frac{d}{dt}(v(t) \cdot w(t)) = \frac{d}{dt}v(t) \cdot w(t) + v(t) \cdot \frac{d}{dt}w(t)$.

product rule for cross product $\frac{d}{dt}(v(t) \times w(t)) = \frac{d}{dt}v(t) \times w(t) + v(t) \times \frac{d}{dt}w(t)$.

chain rule $\frac{d}{dt}F(r(t)) = \nabla F(r(t)) \cdot r'(t)$ (not subject of this midterm).