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Book: Multivariable calculus by J. Stewart

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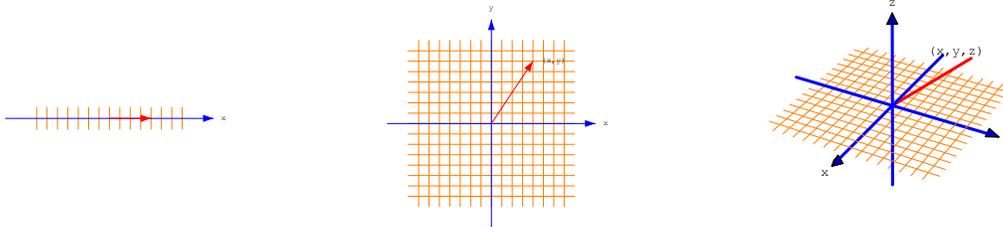
Homework: Due Thursdays

Due 9/26: **9.1: 5, 8, 10, 14, 28**

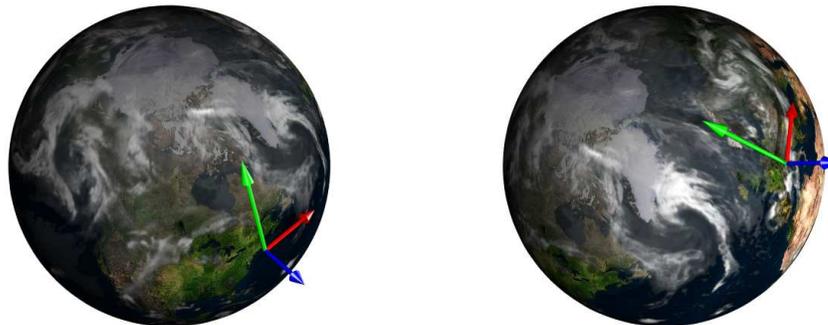
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CARTESIAN COORDINATE SYSTEMS. Points on the line are labeled by 1 coordinate x , points in the **plane** are labeled by 2 coordinates (x, y) . Points in space are determined by three coordinates (x, y, z) .

1D space = line = 2 half lines 2D space = plane = 4 quadrants 3D space = space = 8 octants



CHOICE
OF COOR-
DINATE
SYSTEM



The directions of the x , y or z -coordinate axes determine one of many possible coordinate systems. On earth for example, the z -axes usually points up. But this direction of course depends on the place.

QUESTION: in 3D graphics (computer games, virtual reality, ray tracing) it is custom to have the y axis is up, the x axis to the right and the z axis in front. This is the "photographers" point of view". The photograph is the x - y plane, the depth is the z axes. Is this a left or righthanded coordinate system?

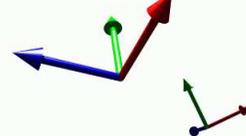
(Remark: The area in graphics memory reserved for storing the z -axis is called the "z-buffer". This is useful for "hidden line removal" in 2D rendering of a 3D scene. The z -axis is perpendicular to the screen with values increasing towards the viewer. Any point whose z coordinate is less than the corresponding z -buffer value will be hidden behind some feature which has already been plotted.)

DISTANCE. The distance between two points $P = (x, y, z)$ and $Q = (a, b, c)$ is

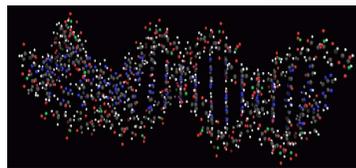
$$d(P, Q) = \sqrt{(x - a)^2 + (y - b)^2 + (z - c)^2} .$$

This can be derived from Pythagoras.

PARITY. We usually work with a right handed coordinate system. The the "right hand rule": thumb= x -direction index finger= y -direction and middle finger= z -direction to check that the coordinate system is "right handed".



Parity plays a role in Biology (orientation of DNA or Proteins) or particle physics, ("parity violation": physical laws are not the same when we look at them in the mirror).



GEOMETRICAL OBJECTS. **curves**, **surfaces** and **bodies** are examples of geometrical objects which can be described using **functions of several variables**. We look at some of them here to get some feel about space. The objects will be treated later in more detail.

SPHERE. The sphere is the collection of points which have a fixed distance r from a given point (a, b, c) . The equation is $(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$. **ICE** Completion of square.

PROBLEM. Draw all points (x, y) which satisfy $x \geq 3$ and $y \leq 2$. SOLUTION. It is a quadrant with corner at $(3, 2)$.

PROBLEM. The coordinate axes $x = 0$, $y = 0$ divide the plane into 4 regions called quadrants. Similarly, the coordinate planes $x = 0$, $y = 0$ and $z = 0$ divide the space into 8 regions called octants. How many "hyperregions" are there in four dimensional "hyperspace" which is labeled by points with 4 coordinates (t, x, y, z) ? Give a point for each of these hyperregions.

SOLUTION. There are 16 regions. Points are $(\pm 1, \pm 1, \pm 1, \pm 1)$

IN CLASS EXERCICE (ICE). Tracking by PDA's. (Triangularisation).

PROBLEM. We want to draw the set of all points (x, y, z) which satisfy $x + 2y - 3z = 2$.
 a) One way to do this is to figure out, where the set intersects the x -coordinate axes, the y -coordinate axes and the z -coordinate axes and put a plane through these three points. Find these intersection points.
 b) An other way to visualize the set is to find the traces, the intersections with the coordinate planes $x = 0$, $y = 0$ or $z = 0$. Find these traces.
 c) Make a drawing of the plane which shows the intercepts found in a) and the traces found in b).

PROBLEM. We have been drawing the coordinate axes in a particular way. Other coordinate axes can be obtained by rotation. Draw a coordinate system which can not be turned into the usual coordinate system.

SOLUTION. Use the left instead of the right hand in the "right hand rule".

HISTORICAL. René Descartes (1596-1650) is credited for introducing the Cartesian coordinate system.

Anectote: "In 1649 Queen Christina of Sweden persuaded Descartes to go to Stockholm. However the Queen wanted to draw tangents at 5 a.m. and Descartes broke the habit of his lifetime of getting up at 11 o'clock. After only a few months in the cold northern climate, walking to the palace for 5 o'clock every morning, he died of pneumonia.

