VECTORS. Two points $P_1 = (x_1, y_1, z_1)$, $Q = P_2 = (x_2, y_2, z_2)$ determine a **vector** $v = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$. It points from P_1 to P_2 and we can write $P_1 + v = P_2$.
Points P in space are in one to one correspondence to vectors pointing from 0 to P. The numbers v_i in a vector $v = (v_1, v_2, v_3)$ are also called **coordinates** of the vector.

REMARK: vectors can be drawn **everywhere** in space. If a vector starts at 0, then the vector $v = (v_1, v_2, v_3)$ points to the point (v_1, v_2, v_3). That's is why one can identify points $P = (a, b, c)$ in space with a vector $v = (a, b, c)$. Two vectors which are translates of each other are considered equal. (*[1])

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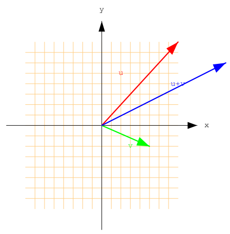
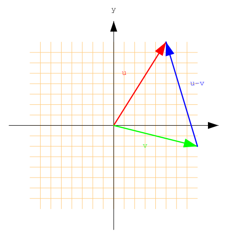
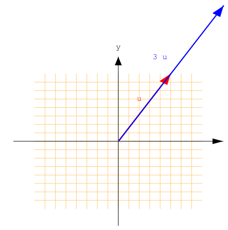
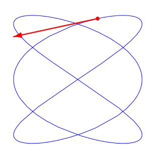
ADDITION SUBTRACTION, SCALAR MULTIPLICATION.

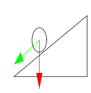
BASIS VECTORS. The vectors $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$ and $\mathbf{k} = (0, 0, 1)$ are called **basis vectors**.

Every vector $v = (v_1, v_2, v_3)$ can be written as a sum of basis vectors: $v = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$.

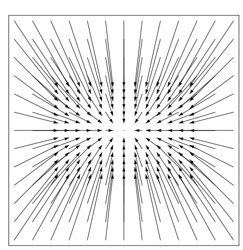
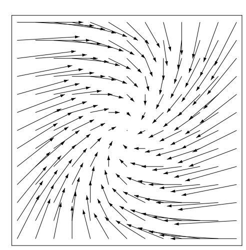
WHERE DO VECTORS OCCUR?
Velocity: if $(f(t), g(t), h(t))$ is a curve, then $v = (f'(t), g'(t), h'(t))$ is the **velocity vector** at the point $(f(t), g(t), h(t))$.



Forces: static problems involve the determination of a force on objects.

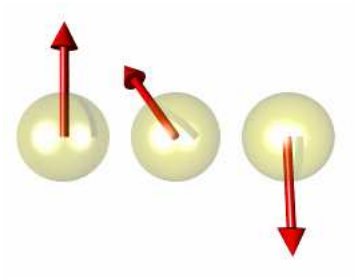


Fields: fields like electromagnetic or gravitational fields or velocity fields in fluids are described with vectors.

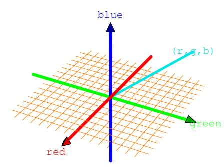



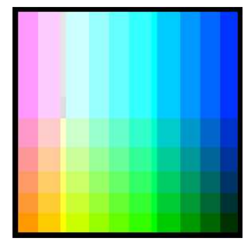
[1] The remark is a common point of confusion. Mathematicians call vectors **affine vectors** and restrict the word **vectors** to affine vectors attached to zero. Calculus courses don't want to add too much terminology and call affine vectors simply **vectors**. Sometimes, vectors attached to 0 are called **bound vectors**. Most courses (like this one) opt for more simplicity and use only the word **vectors** - considering the resulting confusion not grave enough to worry about.

Qbits: in quantum computation, one does not work with bits, but with qbits, which are vectors.

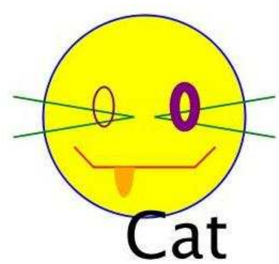


Color Any color can be written as a vector $v = (r, g, b)$, where r is the red component, g is the green component and b is the blue component.





SVG Scalable Vector Graphics is an emerging standard for the web. From www.w3.org: "SVG is a language for describing two-dimensional graphics in XML. SVG allows for three types of graphic objects: vector graphic shapes (e.g., paths consisting of straight lines and curves), images and text. Graphical objects can be grouped, styled, transformed and composited into previously rendered objects. The feature set includes nested transformations, clipping paths, alpha masks, filter effects and template objects.



VECTOR OPERATIONS: The addition and scalar multiplication of vectors satisfies some properties. They are all "obvious" (there is no point in memorizing them).

$u + v = v + u$	commutativity
$u + (v + w) = (u + v) + w$	additive associativity
$u + 0 = u + 0$	null vector
$r * (s * v) = (r * s) * v$	scalar associativity
$(r + s)v = v(r + s)$	distributivity in scalar
$r(v + w) = rv + rw$	distributivity in vector
$1 * v = v$	one element

LENGTH OF A VECTOR.
The length $||v||$ of a vector v is the distance from the beginning to the end of the vector.
EXAMPLE. The length of the vector $v = (3, 4, 5)$ is $||v|| = \sqrt{50} = 5\sqrt{2}$.

TRIANGLE INEQUALITY: $||u + v|| \leq ||u|| + ||v||$.

UNIT VECTOR.
A vector of length 1 is called a **unit vector**. If v is a vector which is not zero, then $v/||v||$ is a unit vector.
EXAMPLE: If $v = (3, 4)$, then $v = (2/5, 3/5)$ is a unit vector in the plane.

PARALLEL VECTORS.
Two vectors v and w are called **parallel**, if $v = rw$ with some constant w.