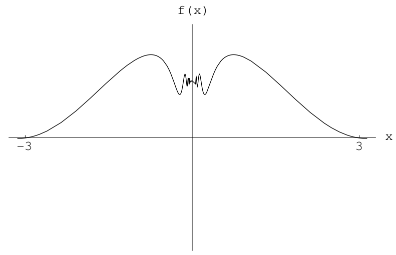
GRAPHS. If $f(x)$ is a function of one variable, then $\{(x, f(x))\}$ is a graph. Graphs are examples of curves in the plane but form a rather narrow class of curves.

EXAMPLE 1. Let $f(x) = \cos(x) + x\sin(x)$ defined on $[-\pi, \pi]$. The graph of f is shown in the picture to the right.



PARAMETRIC CURVES. If $f(t)$, $g(t)$ are functions in one variable, defined on some **parameter interval** $I = [a, b]$, then $r(t) = (f(t), g(t))$ is a **parametric curve** in the plane. The functions $f(t), g(t)$ are called **coordinate functions**. Often, especially in physical context, one write $x(t) = f(t)$ and $y(t) = g(t)$.

EXAMPLE 2. If $x(t) = t$, $y(t) = t^2 + 1$, we can write $y(x) = x^2 + 1$ and the curve is a **graph**.

EXAMPLE 3. If $x(t) = \cos(t), y(t) = \sin(t)$, then $r(t)$ is a **circle**.

If $x(t), y(t), z(t)$ are functions, then $r(t) = (x(t), y(t), z(t))$ describes a **curve** in space.

EXAMPLE 4. If $x(t) = \cos(t), y(t) = \sin(t), z(t) = t$, then $r(t)$ describes a **spiral**.

IDEA: Think of the **parameter** t as **time**. For every fixed t, we have a point $(x(t), y(t), z(t))$ in space. As t varies, we move along the curve.

EXAMPLE 5. If $x(t) = \cos(2t), y(t) = \sin(2t), z(t) = 2t$, then we have the same curve as in EXAMPLE 4 but we traverse it **faster**. The **parametrisation** changed.

EXAMPLE 6. If $x(t) = \cos(-t), y(t) = \sin(-t), z(t) = -t$, then we have the same curve as in EXAMPLE 4 but we traverse it in the **opposite direction**.

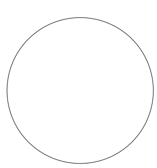
EXAMPLE 7. If $P = (a, b, c)$ and $Q = (u, v, w)$ are points in space, then $r(t) = (a+t(u-a), b+t(v-b), c+t(w-c))$ defined on $t \in [0, 1]$ is a **line segment** connecting P with Q.

ELIMINATION: Sometimes, it is possible to eliminate the variable t and write the curve using equations (one equation in the plane or two equations in space).

EXAMPLE: (circle) If $x(t) = \cos(t), y(t) = \sin(t)$, then $x(t)^2 + y(t)^2 = 1$.

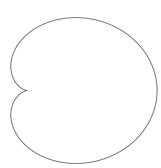
EXAMPLE: (spiral) If $x(t) = \cos(t), y(t) = \sin(t), z(t) = t$, then $x = \cos(z), y = \sin(z)$. The spiral is the intersection of two graphs $x = \cos(z)$ and $y = \sin(z)$.

CIRCLE



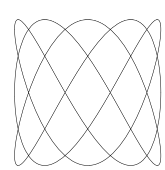
$(\cos(t), 3\sin(t))$

HEART



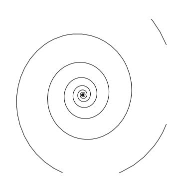
$(1 + \cos(t))(\cos(t), \sin(t))$

LISSAJOUS

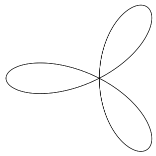
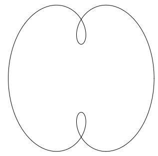
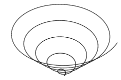
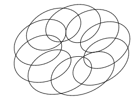


$(\cos(3t), \sin(5t))$

SPIRAL



$e^{t/10}(\cos(t), \sin(t))$

TRIFOLIUM	EPICYCLE	SPRING (3D)	TORAL KNOT (3D)
			
$-\cos(3t)(\cos(t), \sin(t))$	$(\cos(t) + \cos(3t)/2, \sin(t) + \sin(3t)/2)$	$(\cos(t), \sin(t), t)$	$(\cos(t) + \cos(9t)/2, \sin(t) + \cos(9t)/2, \sin(9t)/2)$

WHY DO WE LOOK AT CURVES?
Particles, bodies, quantities changing in time. Examples: motion of a star in a galaxy. Data changing in time like (DJIA(t),NASDAQ(t),SP500(t))

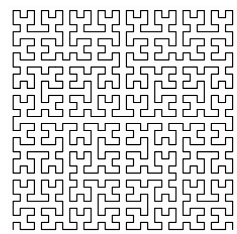
Strings or knots are curves which are important in theoretical physics. Knots are closed curves in space.

Complicated **molecules** like RNA or proteines can be modeled as curves.

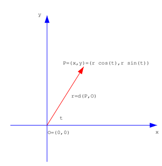
Computergraphics: surfaces are represented by mesh of curves. Representing smooth curves efficiently is important for fast rendering of scenes. The progress in this field is not only due to better computers but also due to mathematics.

Space time A curve in space-time describes the motion of particles. In general relativity gravity is described through curves in a curved space time.

Curves are also interesting in **topology** (i.e. peano curves, boundaries of surfaces, knots).



POLAR COORDINATES. A point (x, y) in the plane has the **polar coordinates** $r = \sqrt{x^2 + y^2}, \theta = arctg(y/x)$. We have $x = r\cos(\theta)$, $y = r\sin(\theta)$.



POLAR CURVES. A general polar curve is written as $(r(t), \theta(t))$. It can be translated into x, y coordinates: $x(t) = r(t)\cos(\theta(t), y(t) = r(t)\sin(\theta(t))$.
POLAR GRAPHS. Curves which are graphs when written in polar coordinates are called **polar graphs**.
EXAMPLE 8. $r(\theta) = \cos(3\theta)$ is the **trifoil** which belongs to the class of **roses** $r(t) = \cos(nt)$.

EXAMPLE 9. If $y = 2x + 3$ is a line, then the equation gives $r\sin(\theta) = 2r\cos(\theta) + 3$. Solving for $r(t)$ gives $r(\theta) = 3/(\sin(\theta - 2\cos(\theta))$. The line is also a polar graph.