CURVES. A vector-valued map on the real line $r(t) = (x(t), y(t))$ or $r(t) = (x(t), y(t), z(t))$ is called a **curve**.
EXAMPLES.

1) $r(t) = (\cos(t), \sin(t))$ is a circle in the plane.
2) $r(t) = (\cos(t), \sin(t), t)$ is a spiral in space.
3) $r(t) = P + tv = (P_1 + tv_1, P_2 + tv_2, P_3 + tv_3)$ is a straight line connecting P $(t = 0)$ with Q $(t = 1)$.

DERIVATIVES. If $r(t) = (x(t), y(t), z(t))$ is a vector valued function describing a curve, then

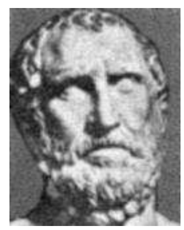
$$r'(t) = (x'(t), y'(t), z'(t)) = (\dot{x}, \dot{y}, \dot{z})$$

is called the **derivative** of r. (The notation with the dot is common, when the parameter is time.) The derivative is also called the **velocity**. The length of the velocity vector is called the **speed**. The derivative of the velocity is called **acceleration**. While the velocity vector is tangent to the curve, the acceleration can point in any direction.

EXAMPLE. If $r(t) = (\cos(3t), \sin(2t), 2\sin(t))$, then $r'(t) = (-3\sin(3t), 2\cos(2t), 2\cos(t))$.

WHAT IS MOTION?

The paradoxon of Zeno of Elea: "If we look at a body at a specific time, then the body is fixed. Having it fixed at each time, there is no motion". While one might wonder today a bit about Zeno's naivity, there were philosophers in our time like Kant, Hume or Hegel, who thought about Zeno's paradoxons. Also physisists continue to ponder about the question what is time and space.



WHAT IS A DERIVATIVE?

The derivative or rate of change is a **limit**. It can be approximated by the vector $(r(t+dt) - r(t))/dt$, where dt is a small number. If dt approaches zero, and the limit exists, the velocity exists at this point. If $r(t) = P + vt$ is a line, then $r'(t) = v$.

EXAMPLES.
1) If $r(t) = P + vt$ is a line, then $r'(t) = v$.
2) If $r(t) = (|t|, t^2, \sqrt{t+1})$, then $r'(t) = (\text{sign}(t), 2t, 2t/\sqrt{t+1})$. The derivative exists at all times except at $t = 0$ and $t = -1$.

EXAMPLES OF VELOCITIES.

Electrons in Metals:	0.005 m/s
Person walking:	1.5 m/s
Car:	15-50 m/s
Signals in nerves:	40 m/s
Aeroplane:	70-900 m/s
Sound in air:	Mach1=340 m/s
Satellite:	1200 m/s
Speed of bullet:	1200-1500 m/s
Earth around the sun:	30'000 m/s
Sun around galaxy center:	200'000 m/s
Light in vacuum:	300'000'000 m/s

EXAMPLES OF ACCELERATIONS.

Train:	0.1-0.3 m/s^2
Car:	3-8 m/s^2
Space shuttle:	$\leq 3G = 30m/s^2$
Combat plane (F16) (blackout):	9G=90 m/s^2
Ejection from F16:	14G=140 m/s^2.
Free fall:	1G = 9.81 m/s^2
Electron in vaccum tube:	10^{15} m/s^2

INTEGRATION. If $v(t) = (x(t), y(t), z(t))$ is a curve, then $\int_0^t v(t) \; dt$ is defined as $(\int_0^t x(t)\,dt, \int_0^t y(t)\,dt, \int_0^t z(t)\,dt)$.

APPLICATION. A flight recorder in a space object records the accelerations $(a(t), b(t), c(t) = (\sin(2t), \sin(t), t)$ in x, y, z direction. The accelerations are accessible because they are proportional to forces, the device can measure. If the plane is at rest at $(0,0,0)$ when $t = 0$, where is it at $t = 10\pi$?

ANSWER. We know $r''(t) = (\cos(t), \sin(t), t)$. By integration, we obtain $\int_0^t r''(t) \; dt = r'(t) = (\cos(t)/2, -\cos(t), t^2/2)$ and $r(t) = (-\sin(2t)/4, -\sin(t), t^3/6)$. At $t = 10\pi$, we have $r(10\pi) = (0, 0, 1000\pi^3/6)$.

ARC LENGTH. If $r(t)$ is a curve defined on some parameter interval $[a, b]$, and $v(t) = r'(t)$ is the velocity and $||v(t)||$ is the speed, then $\int_a^b ||v(t)||\,dt$ is called the **length of the curve**.

Written out, the formula is

$$L = \int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\; dt \; .$$

EXAMPLE. Let $r(t) = (\cos(t), \sin(t), t)$ be a spiral defined for $t \in [0, 10\pi]$ then $||v(t)|| = \sqrt{2}$ and the length is $10\pi\sqrt{2}$.

REMARK. Often we will not be able to find a closed formula for the lenght of a curve. For example the Lissajoux figure $r(t) = (\cos(3t), \sin(5t))$ has length $\int_0^{2\pi} \sqrt{9\sin^2(3t) + 25\cos^2(5t)}\; dt$ which can be evaluated numerically only.

NEWTONS LAW.

Newton second law says
$m\ddot{t} = mr''(t) = F(t)$
where $F(t)$ is the external force acting on the body and m is the **mass** of the body.

GRAVITY. If $F(t) = (0, 0, -gm)$, then a body feels a constant acceleration towards the ground:

$$\ddot{r} = -g = -9.81 m/s^2 \; .$$

QUESTION. If we drop a body from height h, how long does it take to hit the ground?

ANSWER. The position at time t is $(0, 0, h - gt^2/2)$. For $t = \sqrt{2h/g}$, we are at the ground. For example, if $h = 10$ meters, then we have wait about 1.4 seconds.

PROBLEM. In the movie "Six days, seven nights" (with Harrison Ford), pirats shoot with a cannon onto the plane of the heros. They aim however vertically up onto the plane. The 25mm bullet has an initial speed of 35m/s. How much time do the pirats have until the boat is hit by their own bullet? Assume $g = 10$.
ANSWER. If $v(t)$ is the speed of the cannon shell, then $v'(t) = -g$ and $v(t) = 35 - 10 * t$. The velocity is zero after 3.5 seconds. The pirats have therefore 7 seconds to leave the boat.

HISTORY OF NEWTON'S LAW.

Ancient Greek philosophers thought that the motions of the stars and planets were unrelated to events on the earth. The understanding of gravity changed with Galileo, Kepler, Brahe and Newton in the 16'th century. Galileo realized that the gravitational acceleration is independent of the mass of the body. By 1666 Newton did not understand the mechanics of circular motion yet. In 1666 he imagined that the Earth's gravity is influenced the Moon, counterbalancing its centrifugal force. From his law of centrifugal force and Kepler's third law of planetary motion, Newton deduced the inverse-square law. In 1679 Newton corresponded with Hooke who had written to Newton claiming "... that the attraction always is in a duplicate proportion to the distance from the center reciprocall". But Newton then himself derived Kepler's laws from the law of central forces. (See Book: "Huygens and Barrow, Newton and Hooke" by V.I. Arnold) No portrait survives of Robert Hooke. His name is somewhat obscure today, due in part to the enmity of his famous, influential, and extremely vindictive collegue Sir Isaac Newton.