Name:

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MWF 12 Stepan Paul
MWF 12 Hunter Spink
MWF 12 Nathan Yang
MWF 1:30 Fabian Gundlach
MWF 1:30 Flor Orosz-Hunziker
MWF 3 Waqar Ali-Shah

- Start by printing your name in the above box and please **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1,2,8 and 9, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, slide rules, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) (20 points) No justifications are needed. Check T if true and F if false.

1)	T	F	The plane $7x + 9y + 3z = 18$ intersects the z-axis in the point $(0, 0, 3)$.
2)	T	F	For two vectors $\vec{v}$ and $\vec{w}$, the dot product $\vec{v} \cdot \vec{w}$ is always a real number in the interval $[0, 1]$.
3)	T	F	The area of any parallelogram spanned by two vectors $\vec{u}, \vec{v}$ is given by the formula $\|\vec{u}\|\|\vec{v}\|$.
4)	T	F	Given two points A, B in the plane $\mathbf{R^2}$, there is at least one point C such that $\|\vec{AC}\| = \|\vec{BC}\|$.
5)	T	F	The curve $\vec{r}(t) = [1 + t^6, 2 + 2t^6, 3 - 3t^6]$ is a line.
6)	T	F	The surface $-x^2 - y + y^2 = z$ is a hyperbolic paraboloid = saddle surface.
7)	T	F	If $\vec{v} \cdot \vec{w} = 0$, then the angle between two non-zero vectors $\vec{v}$ and $\vec{w}$ is $\pi/2$.
8)	T	F	If the velocity of $\vec{r}(t)$ is a constant non-zero vector $\vec{v}$ for all t, then the curve is a line.
9)	T	F	The parametric curves $\vec{r}_1(t) = [0, t, -t]$ and $\vec{r}_2(s) = [1 + s, 1 + s, 0]$ do not intersect.
10)	T	F	The equation $\cos^2(\rho) = 1/\sqrt{2}$ given in spherical coordinates defines a sphere.
11)	T	F	If $\|\vec{v} \times \vec{w}\| = 0$ and $\vec{v}$ and $\vec{w}$ are unit vectors, then $\vec{v} = \vec{w}$.
12)	T	F	The surface given in cylindrical coordinates as $\cos(\theta) + \sin(\theta) = r$ is a cylinder.
13)	T	F	The arc length of a curve $\vec{r}(t)$ with $0 \leq t \leq 1$ is $\| \int_0^1 \vec{r}\,'(t)\, dt\|$.
14)	T	F	There is a time t, when the velocity vector of $\vec{r}(t) = [\cos(t), \sin(t), t]$ is parallel to the vector $[0, 0, 1]$.
15)	T	F	It is possible that the intersection of two ellipsoids is a parabola.
16)	T	F	If L, M are two parallel lines, then the distance between L and M is the distance of a point P on L to M.
17)	T	F	The line $\vec{r}(t) = [3t, 4t, 6t]$ is perpendicular to the plane $3x + 4y + 6z = 12$.
18)	T	F	Let $\vec{j} = [0, 1, 0]$. There is a vector $\vec{v}$ for which the vector projection of $\vec{v}$ onto $\vec{j}$ is equal to $-\vec{j}$.
19)	T	F	None of the level sets (i.e. contours) of the graph of a hyperbolic paraboloid (i.e. saddle) are lines.
20)	T	F	For the function $f(x, y) = \sin(x) + y$ the level curves (i.e. contour curves) $f(x, y) = 1$ and $f(x, y) = 2$ do not intersect.

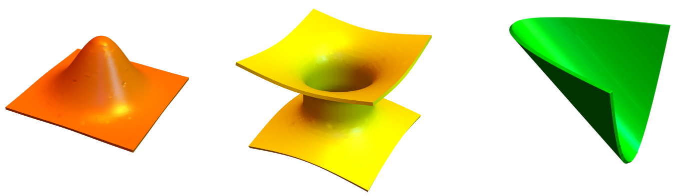
Total

Problem 2) (10 points) No justifications are needed in this problem.

In each sub-problem, each of the numbers 0,1,2,3 each occur exactly once. If all 4 are correct, the score is 2 points, if at least 2 are correct, then the score is 1 point.

a) (2 points) Match the surfaces $g(x, y, z) = c$. Enter 0 if there is no match.

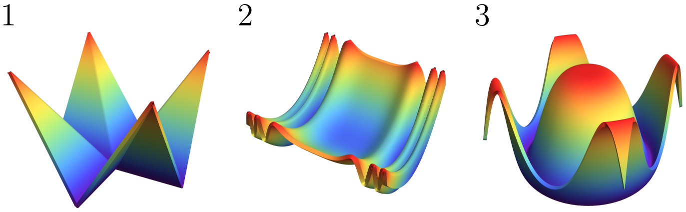
1 2 3



Function $g(x, y, z) =$	0,1,2, or 3
$x - y^2 + z = 0$	
$x^2 + y^2 - z^4 = 1$	
$z - e^{-x^2-y^2} = 0$	
$y^2 + z^3 = 1$	

b) (2 points) Match the graphs of the functions $f(x, y)$. Enter 0 if there is no match.

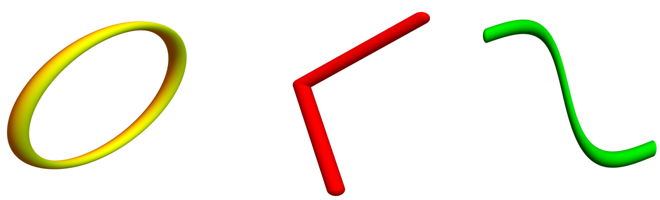
1 2 3



Function $f(x, y) =$	0,1,2, or 3
$\cos(x^2 + y^2)$	
$\|\|x - y\| - \|x + y\|\|$	
$y^2 \sin(x^4)$	
$\sqrt{\|1 + x^2 - y^2\|}$	

c) (2 points) Match the space curves with the parametrizations. Enter 0 if there is no match.

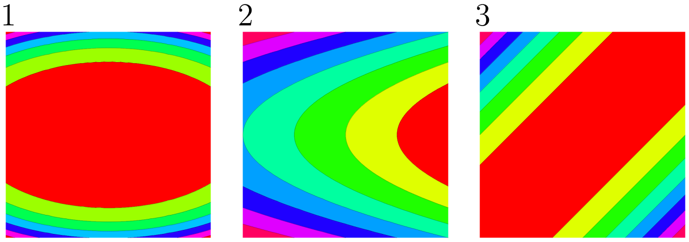
1 2 3



Parametrization $\vec{r}(t) =$	0,1,2, or 3
$[\cos(2t), 0, \cos(2t)]$	
$[0, \cos(2t), \sin(2t)]$	
$[t, \sin(t), 0]$	
$[\|t\|, t, t]$	

d) (2 points) Match the functions g with contour plots in the xy-plane. Enter 0 if there is no match.

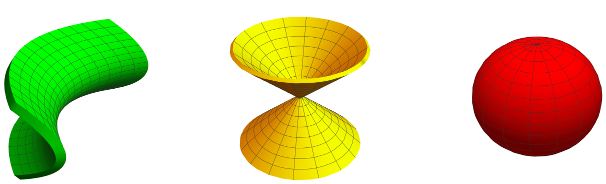
1 2 3



Function $g(x, y) =$	0,1,2, or 3
$(x - y)^2$	
$y^2 - x$	
$\cos(2x) + \sin(2y)$	
$(2x^2 + 7y^2)^2$	

e) (2 points) Match the parametrized surfaces. Enter 0 if there is no match.

1 2 3



Surface $\vec{r}(u, v)$	0-3
$[\sin(v)\cos(u), \sin(v)\sin(u), \cos(v)]$	
$[u^2 - v^2, u, v]$	
$[u, u^2 + v^2, v]$	
$[u\cos(v), u\sin(v), u]$	

Problem 3) (10 points)

We perform some computations with the vectors $\vec{v} = [2, 2, 2]$ and $\vec{w} = [3, 2, 1]$.

a) (2 points) Construct a unit vector parallel to $\vec{v}$.

b) (2 points) What is the cross product $\vec{v} \times \vec{w}$?

c) (2 points) Find $\cos(\alpha)$ of the angle α between $\vec{v}, \vec{w}$.

d) (2 points) What is the vector projection $\vec{P}_{\vec{w}}(\vec{v})$ of $\vec{v}$ onto $\vec{w}$?

e) (2 points) What is the scalar projection of $\vec{v}$ onto $\vec{w}$?

Problem 4) (10 points)

The **global positioning system** uses distances to satellites to determine the position of a point. Assume three satellites are at $A = (1,1,1), B = (1,2,3), C = (4,2,1)$.

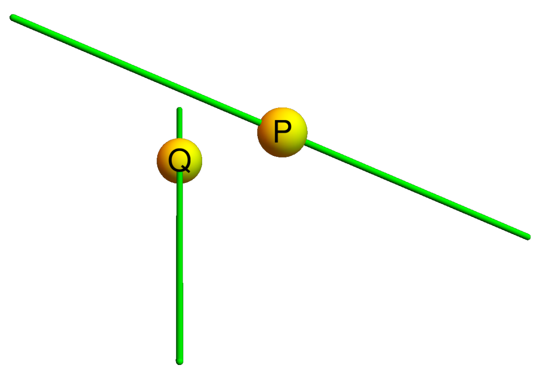
a) (4 points) What is the area of the triangle ABC?

b) (3 points) Find the equation $ax + by + cz = d$ of the plane through A and B and C.

c) (3 points) Write down a parametrization $\vec{r}(s,t) = [x(s,t), y(s,t), z(s,t)]$ of this plane such that $\vec{r}(0,0) = [1,1,1]$.

Problem 5) (10 points)

Our goal is to compute the distance between the lines L given by $\vec{r}_1(t) = [3+t, 2+t, 4-t]$ and M given by $\vec{r}_2(t) = [2-t, 1+t, 3]$.



a) (2 points) First find a point P and a vector $\vec{v}$ parallel to and contained in L.

b) (2 points) Now find a point Q and a vector $\vec{w}$ parallel to and contained in M.

c) (6 points) Compute the distance between L and M.

Problem 6) (10 points)

Our goal is to compute the arc length of the curve

$$\vec{r}(t) = \left[\ \frac{t^5}{5}, \frac{t^9}{9}, \sqrt{2}\frac{t^7}{7}\ \right]$$

where t satisfies $0 \leq t \leq 1$.

a) (2 points) Write down the velocity $\vec{r}\,'(t)$.

b) (4 points) Compute the speed at time t and simplify.

c) (4 points) What is the arc length of the curve parametrized with $t \in [0,1]$?

Problem 7) (10 points)

You have a ride on the roller coaster **Goliath** at Six Flags New England. Your smartphone measures the acceleration

$$\vec{r}\,''(t) = [\cos(t), -\sin(t), 0].$$

You know also the position and velocity at time $t = 0$:

$$\vec{r}(0) = [0, 0, 5]\,, \vec{r}\,'(0) = [1/2, 1, 0]\,.$$

a) (4 points) Determine $\vec{r}\,'(t)$.

b) (4 points) Compute $\vec{r}(t)$.

c) (2 points) Where are you at $t = \pi$?

Problem 8) (10 points)	(Only answers are needed)

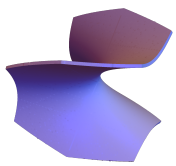
a) (3 points) Assume the vectors $\vec{v}$ and $\vec{w}$ are unit vectors. Please check the corresponding box in each row or none.

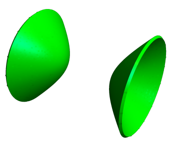
Expression	is always $\vec{0} = [0,0,0]$	can be a nonzero vector
$\vec{v} \times \vec{w}$		
$(\vec{v} + \vec{w}) \cdot (\vec{v} - \vec{w})$		
$\vec{v} \times (3\vec{v})$		

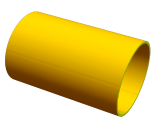
b) (3 points) Please complete the table so that the same surface is described in each row in the corresponding coordinate system. Each correct row gives a point.

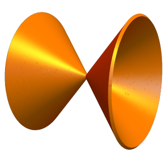
Cartesian coordinate	Cylindrical coordinates	Spherical coordinates
	$r^2 + z^2 = 1$	
		$\phi = \pi/2$
$x^2 + y^2 = 1$		

c) (4 points) The following surfaces are in need of a name. Here is a word bank of possible answers: cylinder, cone, torus, plane, cylindrical paraboloid, ellipsoid, hyperbolic paraboloid, elliptic paraboloid, one sheeted hyperboloid, two sheeted hyperboloid,









Problem 9) (10 points) No justifications are needed.

We eat ice cream at the Kimball Farm in Carlisle. The cup is part of a cone topped off with a parabolic rim, the bottom is a disc, the straw is a cylinder. Please complete the parametrizations. In each part, we have given you one of the coordinates. You should use the variables which are provided to the left.

Kimball farm. (Photo: Bill Meier, 2018)

a) (2 points) The **cup surface** is $x^2 + y^2 = z^2/25$.

$$\vec{r}(r,\theta) = [\ r\cos(\theta)\ ,\ \quad\ ,\ \quad\ ]$$

b) (2 points) The **parabolic rim** $z = 25 + x^2 + y^2$.

$$\vec{r}(x,y) = [\ \quad\ ,\ y\ ,\ \quad\ ]$$

c) (2 points) The **strawberry ice cream** $(x-2)^2 + y^2 + z^2 = 36$.

$$\vec{r}(\theta,\phi) = [\ \quad\ ,\ \quad\ ,\ 6\cos(\phi)\ ,].$$

d) (2 points) The **cookie straw** $x^2 + y^2 = 1$

$$\vec{r}(\theta,z) = [\ \quad\ ,\ \quad\ ,\ z\ ].$$

e) (2 points) The table $z = 15$.

$$\vec{r}(x,y) = [\ x\ ,\ \quad\ ,\ \quad\ ]$$