Name:

MWF 9 Oliver Knill
MWF 9 Arnav Tripathy
MWF 9 Tina Torkaman
MWF 10:30 Jameel Al-Aidroos
MWF 10:30 Karl Winsor
MWF 10:30 Drew Zemke
MWF 12 Stepan Paul
MWF 12 Hunter Spink
MWF 12 Nathan Yang
MWF 1:30 Fabian Gundlach
MWF 1:30 Flor Orosz-Hunziker
MWF 3 Waqar Ali-Shah

- Start by printing your name in the above box and please **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, slide rules, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

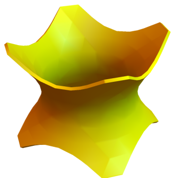
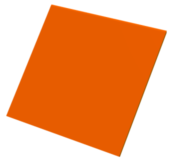
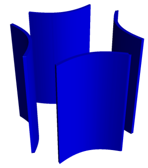
Problem 1) (20 points) No justifications are needed.

1)	T	F	If $|\vec{r}\,'(t)| = 1$ for all t, then the arc length of the curve defined by $\vec{r}(t)$ for $2 \leq t \leq 5$ is equal to 3.
2)	T	F	The length of the vector connecting $A = (3, 4, 5)$ to $B = (6, 8, 5)$ is 5.
3)	T	F	If $|\vec{v} + \vec{w}|^2 = 2\vec{v} \cdot \vec{w}$, then $\vec{v} = 2\vec{w}$.
4)	T	F	The distance between two points P and Q is smaller than or equal to $|\vec{OP}| + |\vec{OQ}|$, where $O = (0, 0, 0)$ is the origin.
5)	T	F	The line $\vec{r}(t) = [5 + 2t, 2 + t, 3 + t]$ is located on the plane $x - y - z = 0$.
6)	T	F	The arc length of a circle of radius r is πr^2.
7)	T	F	The surface $x^2 - y^2 + 4y = z^2 + 2z$ is an elliptic paraboloid.
8)	T	F	For any two vectors, $|\vec{v} \cdot \vec{w}| \leq |\vec{v}| + |\vec{w}|$.
9)	T	F	The acceleration of $\vec{r}(t) = [t, t, t]$ is $[0, 0, 0]$ everywhere.
10)	T	F	If $\vec{v} = \vec{PQ} = [2, 1, 1]$ then $|\vec{v}|$ is larger than the distance between P and Q.
11)	T	F	If $\vec{v} \cdot \vec{w} > 0$, then the angle between $\vec{v}$ and $\vec{w}$ is larger than $90°$.
12)	T	F	If $[5, 6, 4] \times \vec{x} = \vec{x}$ then $\vec{x} = [0, 0, 0]$.
13)	T	F	The curve $\vec{r}(t) = [t - 1, 1 - t, t]$ has constant acceleration $[0, 0, 0]$.
14)	T	F	The line $\vec{r}(t) = t[7, 7, 1]$ hits the plane $-x - y = 7z$ in a right angle.
15)	T	F	The surface given in spherical coordinates as $\sin(\phi) = 1/\rho$ is a cylinder.
16)	T	F	Given three vectors $\vec{u}, \vec{v}$ and $\vec{w}$, then $|(\vec{u} \cdot \vec{v})(\vec{w} \cdot \vec{u})| \leq |\vec{u}|^2 |\vec{v}||\vec{w}|$.
17)	T	F	The surface given in spherical coordinates as $\rho^2 = 1$ is a sphere.
18)	T	F	The arc length of the curve $[5\sin(t), 1, 5\cos(t)]$ from $t = 0$ to $t = 2\pi$ is equal to 10π.
19)	T	F	The surface parametrized by $\vec{r}(u, v) = [u^5 - v^5, u^5 + v^5, u^5]$ is a plane.
20)	T	F	It can happen that the cross product of two non-zero vectors $\vec{v}$ with $\vec{w}$ is parallel to $\vec{v}$.

	Total

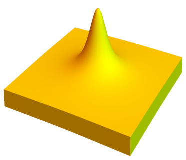
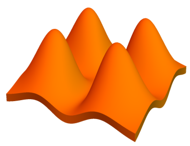
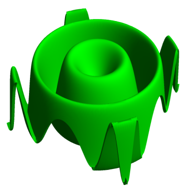
Problem 2) (10 points) No justifications are needed in this problem.

a) (2 points) Match the contour surfaces $g(x, y, z) = 1$. Enter O, if there is no match.

I

II

III


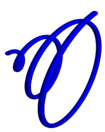
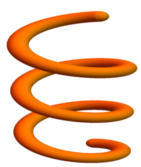
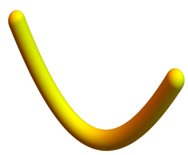
Function $g(x, y, z) = 1$	Enter O,I,II or III
$x^2 + y^2 - z^2$	
$x^2 y^2$	
$x - y$	
$x^2 + z^2$	

b) (2 points) Match the graphs of the functions $f(x, y)$. Enter O, if there is no match.

I

II

III


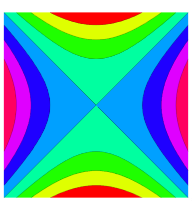
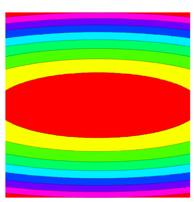
Function $f(x, y) =$	Enter O,I,II or III
$1/(1 + x^2 + y^2)$	
$x^2 y^2 e^{-x^2-y^2}$	
$\sin(x^2 + y^2)$	
$\cos(y)$	

c) (2 points) Match the space curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.

I

II

III


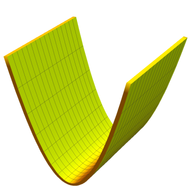
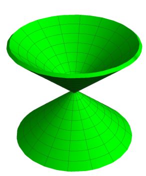
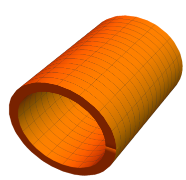
Parametrization $\vec{r}(t) =$	Enter O, I,II or III
$[t, t\sin(3t), t\cos(3t)]$	
$[\cos(t), \sin(t), \cos(t)]$	
$[\sin(t), \cos(t), t]$	
$[t, t, t^2]$	

d) (2 points) Match functions g with contour plots in the xy-plane. Enter O, if there is no match.

I

II

III


Equation	Enter O, I,II or III
$\sin(x) - y$	
$x^2 - y^2$	
$10x^2 + y^2$	
$x^2 + 10y^2$	

e) (2 points) Match the quadrics. Enter O if no match.

I

II

III


Quadric	Enter O,I,II or III
$x^2 = z$	
$x^2 + y^2 = 1$	
$xy = 1$	
$y = 1$	
$x^2 + y^2 = z^2$	

Problem 3) (10 points)	(Only answers are needed)

a) (4 points) Mark what applies for any two vectors $\vec{v}$ and $\vec{w}$ in space.

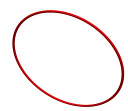
Object	always 0	can be $\neq 0$	always $\vec{0} = [0,0,0]$	can be nonzero vector
$(\vec{v} \times \vec{w}) \times \vec{v}$				
$(\vec{v} \times \vec{w}) \cdot \vec{v}$				
$(\text{proj}_{\vec{v}} \vec{w}) \times \vec{w}$				
$(\text{proj}_{\vec{w}} \vec{v}) \cdot \vec{w}$				

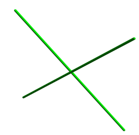
b) (3 points) **Conic sections** (parabolas, ellipses or hyperbolas) can be seen as intersections of a two dimensional cone

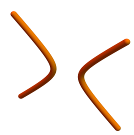
$$x^2 + y^2 = z^2$$

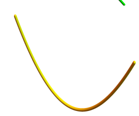
with a 2D plane. Identify the quadrics in the following three cases:

Intersect with plane	Enter A-D
$z = 1$	
$z = \sqrt{2}x$	
$x = y + 1$	

A 

B 

C 

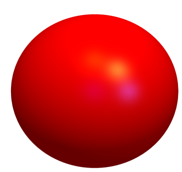
D 

c) (3 points) Three dimensional cone is given by the equation

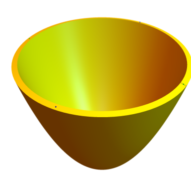
$$x^2 + y^2 + z^2 = w^2$$

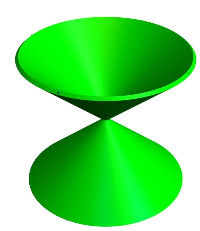
in four dimensional space. If we intersect it with a three dimensional space, we get quadrics. We want you to identify a few quadrics. In the pictures the quadrics might be turned or scaled. You get a point for every right answer, meaning that you can miss one and still have full credit.

Intersect with the 3D plane	Enter A-D or O
$w = 0$	
$w = z + 1$	
$y = z$	
$w = 1$	

A 

B 

C 

D 

Problem 4) (10 points)

We are going into the **furniture design business**. Our first task is to construct a chair. Find the distance between the lines spanned by the parallel arm rests AB and CD, where

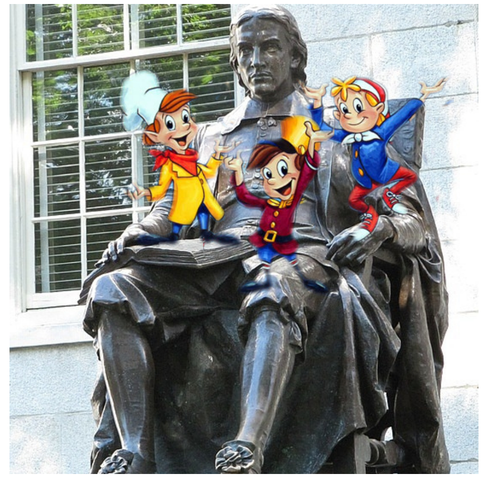
$$A = (1, 0, 2), B = (4, 1, 2)$$

and

$$C = (3, 3, 3), D = (6, 4, 3) .$$

Problem 5) (10 points)

a) (6 points) The first derivative of parametrized curve is called **"velocity"**. You have also learned the terms **"acceleration"** and maybe **"jerk"** for the second and third derivative. Less well known are **"snap"**, **"crackle"**, **"pop"** for the fourth, fifth and sixth derivatives. Since we could not yet find the seventh derivative named, let's call it the **"Harvard"**. Compute the **"Harvard"** of the curve



$$\vec{r}(t) = [\cos(2t) + t, \sin(2t), t^2]$$

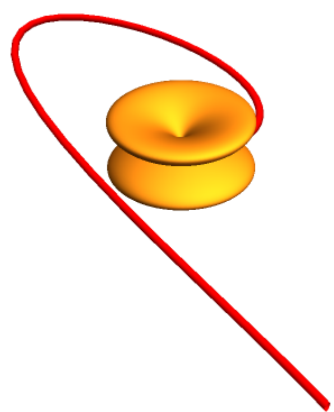
at time $t = 0$.

b) (4 points) The parametrization $\vec{v}(t) = \vec{r}\,'(t)$ defines a new curve. It is located on a surface. Which of the following surface is it?

	Check one
cylinder	A
cone	B
plane	C
ellipsoid	D
paraboloid	E

Problem 6) (10 points)

A kid plays with a **Yo-Yo**. It is accelerated periodically with $\vec{r}\,''(t) = [\sin(t), 0, \cos(t) - 10]$. Find the position of the Yo-Yo at time $t = 2\pi$ if the initial position is



$$\vec{r}(0) = [5, 5, 0]$$

and the initial velocity is

$$\vec{r}\,'(0) = [1, 1, 1] \ .$$

Problem 7) (10 points)

a) (5 points) Compute the arc length of the curve

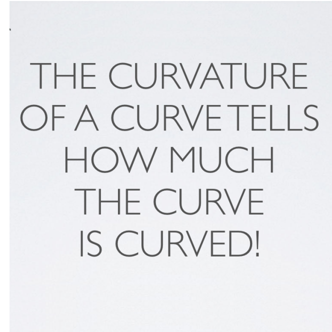
$$\vec{r}(t) = [t, t^2, 2t^3/3]$$

if $0 \leq t \leq 4$.

b) (5 points) What is the following expression

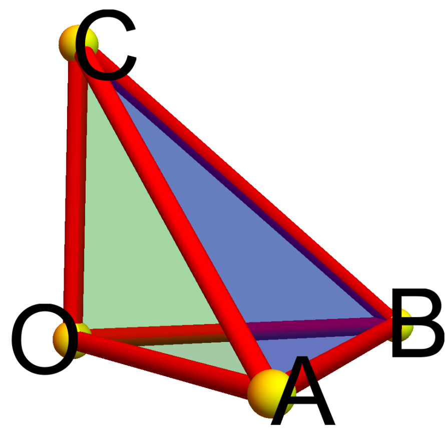
$$\kappa(t) = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$$

called "curvature" of this curve at $t = 0$?



Problem 8) (10 points)

The triangle ABC is obtained by slicing a corner ABC off from a cube. One obtains a so called **trirectangular tetrahedron**.



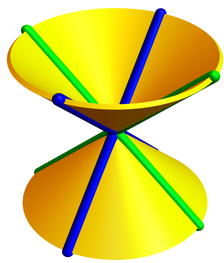
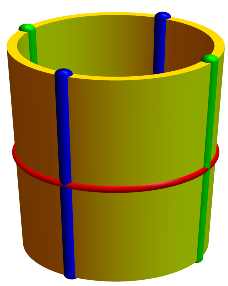
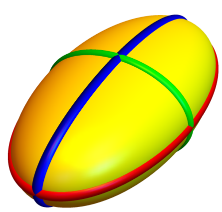
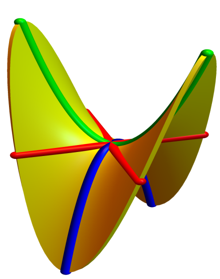
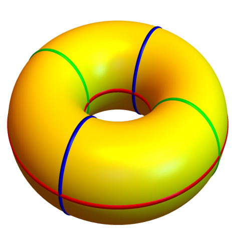
a) (5 points) Find the square of the area of the triangle ABC, where $A = (3, 0, 0), B = (0, 6, 0), C = (0, 0, 8)$.

b) (5 points) Compute also the sum of the squares of the areas of the triangles OAB, OBC and OCA, where $O = (0, 0, 0)$ is the origin. The sum should get the same value you got in a).

P.S. the same computation can be repeated for arbitrary points $A = (a, 0, 0), B = (0, b, 0), C = (0, 0, c)$. It proves a not so well known theorem telling that the sum of the squares of the side wall areas is the square of the face area. It is a 3 dimensional version of Pythagoras and also goes by the name **de Gua theorem** or **Faulhaber theorem**.

Problem 9) (10 points)

The 3D printing venture "Math-Candy" (math-candy.com) asks you to do some product development. In each of the 5 following parametrizations, two entries are still missing, each entry being worth one candy (1 point).

a)		The surface $x^2 + y^2 = z^2$ is parametrized by $\vec{r}(\theta, z) = [\ldots\ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots\ldots, z]$
b)		The surface $x^2 + y^2 = 1$ is parametrized by $\vec{r}(\theta, z) = [\ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots, z]$
c)		The surface $2(x-1)^2 + (y-5)^2 + 4z^2 = 1$ is parametrized by $\vec{r}(\theta, \phi) = [\ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots, \cos(\phi)/2]$
d)		The surface $x^2 - y^2 = z$ is parametrized by $\vec{r}(x, y) = [\ldots\ldots\ldots, \ldots\ldots\ldots, x^2 - y^2]$
e)		The surface $(\sqrt{x^2 + y^2} - 2)^2 + z^2 = 1$ is parametrized by $\vec{r}(\theta, \phi) = [(2 + \cos(\phi))\cos(\theta), \ldots\ldots\ldots\ldots, \ldots\ldots\ldots]$