Name:

MWF 9 Oliver Knill
MWF 9 Arnav Tripathy
MWF 9 Tina Torkaman
MWF 10:30 Jameel Al-Aidroos
MWF 10:30 Karl Winsor
MWF 10:30 Drew Zemke
MWF 12 Stepan Paul
MWF 12 Hunter Spink
MWF 12 Nathan Yang
MWF 1:30 Fabian Gundlach
MWF 1:30 Flor Orosz-Hunziker
MWF 3 Waqar Ali-Shah

- Start by printing your name in the above box and please **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, slide rules, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

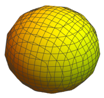
Problem 1) (20 points) No justifications are needed.

1)	T	F	The vector $[2,3,6]$ has a length which is an integer.
2)	T	F	The surface $x^2+y^2+z^2-2x=3$ is a sphere.
3)	T	F	If $\vec{v}\cdot\vec{w}$ is negative, then the angle between $\vec{v}$ and $\vec{w}$ is acute (= smaller than $\pi/2$).
4)	T	F	The level curves $f(x,y)=1$ and $f(x,y)=0$ do not intersect for $f(x,y)=(xy+\cos(x))^6$.
5)	T	F	For any nonzero $\vec{a}$, the equation $\vec{a}\times\vec{x}=\vec{b}$ always has a solution $\vec{x}$.
6)	T	F	For two non-parallel $\vec{a},\vec{b}$, the equation $([x,y,z]\times\vec{a})\cdot\vec{b}=1$ defines a plane.
7)	T	F	The curve $\vec{r}(t)=[4+t^3,1-t^3,5-t^3]$ is a line.
8)	T	F	If $\vec{r}\,(t)\cdot\vec{r}\,(t)=1$, for all t, then $\vec{r}\,'(t)$ is perpendicular to $\vec{r}(t)$ for all t.
9)	T	F	There exist non-parallel vectors $\vec{v},\vec{w}$ such that $\vec{v}\cdot(\vec{v}\times\vec{w})=0$.
10)	T	F	The vector $[3,4,5]$ is perpendicular to the plane $-4x+3y=0$.
11)	T	F	The point given in spherical coordinates as $\rho=3,\phi=\pi/2,\theta=\pi$ is on the x-axes.
12)	T	F	The parametrized curve $\vec{r}(t)=[5\cos(3t),3\sin(3t),0]$ is an ellipse.
13)	T	F	If the vector projection of $\vec{v}$ onto $\vec{w}$ is $\vec{w}$ then $\vec{v}=\vec{w}$.
14)	T	F	Given three vectors $\vec{u},\vec{v}$ and $\vec{w}$, then $|(\vec{u}\times\vec{v})\times\vec{w}|\le|\vec{u}||\vec{v}||\vec{w}|$.
15)	T	F	The surface $y^2+z=x^2$ is a hyperbolic paraboloid.
16)	T	F	The arc length of the curve $[\sin(t),0,\cos(t)]$ from $t=0$ to $t=2\pi$ is equal to 2π.
17)	T	F	The curve $\vec{r}(t)=[\cos(t),\sin(t),\cos(t)+\sin(t)]$ is on the intersection of $x^2+y^2=1$ and $x+y-z=0$.
18)	T	F	Using $\vec{i}=[1,0,0],\vec{j}=[0,1,0]$, the identity $(\vec{i}\times\vec{j})\times\vec{j}=\vec{i}\times(\vec{j}\times\vec{j})$ holds.
19)	T	F	Using the same notation, the identity $(\vec{i}\cdot\vec{j})\vec{j}=\vec{i}(\vec{j}\cdot\vec{j})$ holds.
20)	T	F	$[\cos t,\sin t,t]$, $0\le t\le 2$ and $[\cos(t^3),\sin(t^3),t^3]$, $0\le t\le 2$ have the same arc length.

Total

Problem 2) (10 points) No justifications are needed in this problem.

a) (2 points) Match the contours $g(x,y,z) = 1$. Enter O, if there is no match.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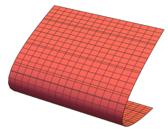
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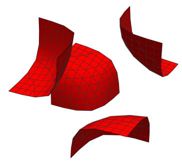
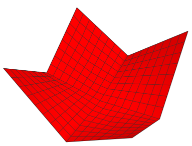

III

Function $g(x,y,z) = 1$	Enter O,I,II or III
xyz	
$x^2 + y^2 + z^2$	
$z^2 - y$	
$x^2 + z^2$	

b) (2 points) Match the graphs of the functions $f(x,y)$. Enter O, if there is no match.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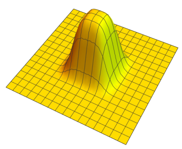
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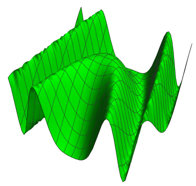
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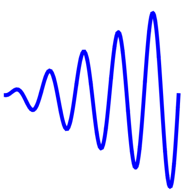
Function $f(x,y) =$	Enter O,I,II or III
$\cos(x^2 + y)$	
$\|x + y\| + \|xy\|$	
$\exp(-x^4 - y^4)$	
x^3	

c) (2 points) Match the plane curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.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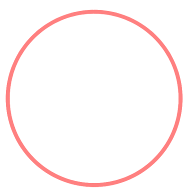
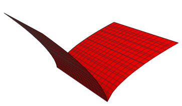

II


III

Parametrization $\vec{r}(t) =$	Enter O, I,II or III
$\vec{r}(t) = [t, t\sin(5t)]$	
$\vec{r}(t) = [t\sin(5t), t]$	
$\vec{r}(t) = [\sin(5t), \cos(5t)]$	
$\vec{r}(t) = [\cos(5t), \cos(5t)]$	

d) (2 points) Match functions g with level surface $g(x,y,z) = 1$. Enter O, if there is no match.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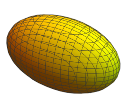
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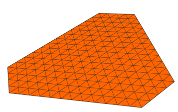
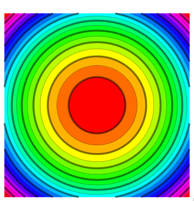

III

Function $g(x,y,z) = 1$	Enter O, I,II or III
$x^2 - y^2 + z^2 = 1$	
$x - y - z = 1$	
$y^3 = z^2$	
$x^2/4 + y^2 + z^2/2 = 1$	

e) (2 points) Match the contour maps to a function $f(x,y)$. Enter O if no match.

I


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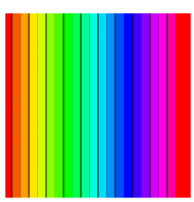

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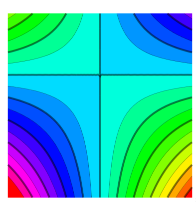
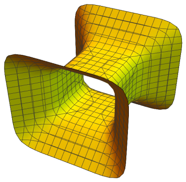


$f(x,y) =$	Enter O,I,II or III
$x^2 - y^4$	
$xy - x$	
x	
y	
$x^2 + y^2$	

Problem 3) (10 points)	(Only answers are needed)

a) (4 points) The following contour surfaces were deformed by setting $X = x^3, Y = y^3, Z = z^3$. Can you label the original quadrics from which it was deformed?



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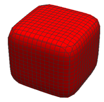

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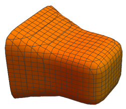

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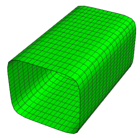


IV

Surface	I-IV	name A-D
$X^2 + Y^2 + Z^2 = 1$		
$X + Y^2 + Z^2 = 1$		
$X^2 - Y^2 + Z^2 = 1$		
$X^2 + Z^2 = 1$		

Fill in:
A) Ellipsoid'esque
B) Paraboloid'esque
C) Hyperboloid'esque
D) Cylinder'esque

b) Help Larry the physicist in the movie "A serious man", to compute some quantities:

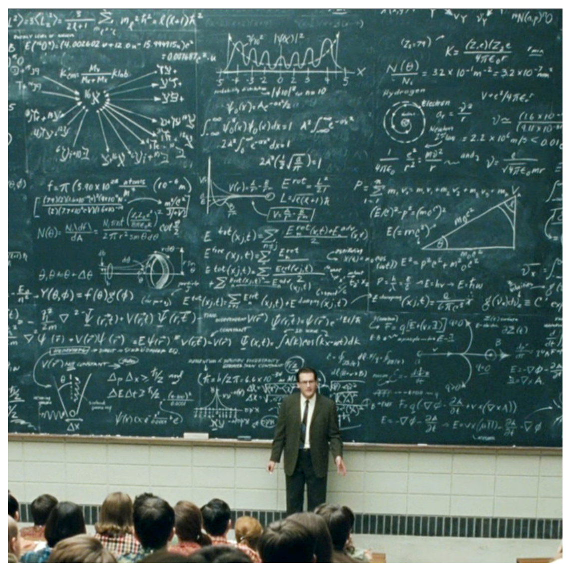
$\vec{v} = [1, 2, 3]$ represent the velocity
$\vec{w} = [0, 1, 1]$ represent angular velocity
$\vec{B} = [1, 0, 1]$ represent magnetic field
$\vec{r} = [0, 0, 1]$ represent position. Compute:

(i) (2 points) Coriolis force $\vec{v} \times \vec{w}$.

(ii) (2 points) Lorentz force $\vec{v} \times \vec{B}$.

(iii) (1 point) Kinetic energy $(\vec{v} \cdot \vec{v})/2$.

(iv) (1 point) Magnetic energy $\vec{B} \cdot \vec{B}/2$.



Larry: "I mean - even I don't understand the dead cat. The math is how it really works."

(i)

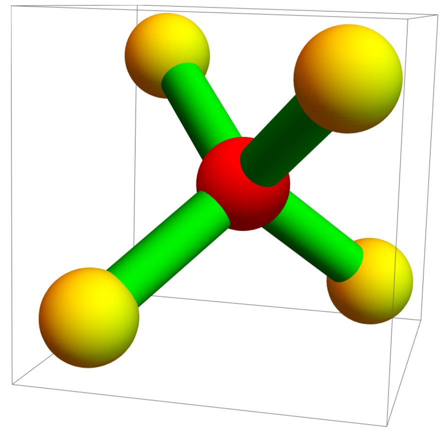
ii)

(iii)

iv)

Problem 4) (10 points)

Methane CH_4 is the number two greenhouse gas emitted by human activity in the US. The four hydrogen atoms of **methane** are located at the vertices $P = (2,2,2), Q = (2,0,0), R = (0,2,0), S = (0,0,2)$ and form a regular tetrahedron, while C is the central carbon atom located at $(1,1,1)$.



a) (2 points) Find one bond distance $|CP|$ and the distance $|PQ|$

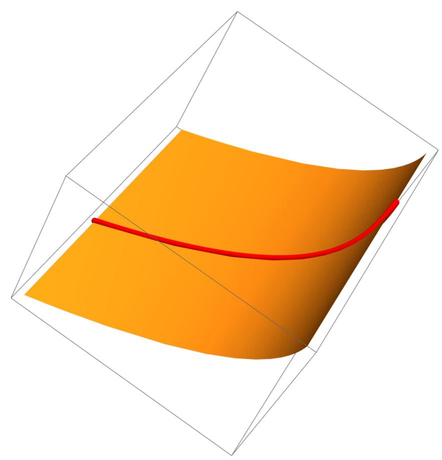
b) (4 points) Find the cosine of the bond angle between $\vec{PC}$ and $\vec{PQ}$.

c) (4 points) What is volume of the parallelepiped spanned by $\vec{PC}, \vec{QC}, \vec{RC}$?

Problem 5) (10 points)

Consider the curve

$$\vec{r}(t) = [2e^t, t, e^{2t}] \ .$$



a) (3 points) Compute the speed $|\vec{r}\,'(0)|$.

b) (5 points) Find the arc length from $t = -2$ to $t = 1$.

c) (2 points) There exists a constant a such that the curve lies on the cylindrical paraboloid $x^2 = az$. Which a does apply?

Problem 6) (10 points)

The highest **bungee jump** ever recorded was done from the 233 meter high Macau Tower. Assume the rope pulls back with a force $2t$ so that the acceleration is

$$\vec{r}\,''(t) = [0, 0, 2t - 10] \ .$$

Assume the initial velocity is $[1, 0, 0]$ and that the daredevil jumps from $\vec{r}(0) = [0, 0, 233]$:

a) (5 points) Find $\vec{r}\,'(t)$ and determine t_0 for which the third component $z'(t_0) = 0$. This is the time of the lowest point.

b) (5 points) Find $\vec{r}(t) = [x(t), y(t), z(t)]$ and $\vec{r}(t_0)$. Did the jumper hit the ground $z = 0$?

Problem 7) (10 points)

The **logarithmic spiral** is parametrized by $\vec{r}(t) = [e^t \cos(t), e^t \sin(t), 0]$.

a) (5 points) Find the angle between $\vec{r}\,'(t)$ and acceleration $\vec{r}\,''(t)$ at time $t = 0$.

b) (5 points) Compute the following expression at $t = 0$.

$$\kappa(t) = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3} \ .$$

It is called the curvature.

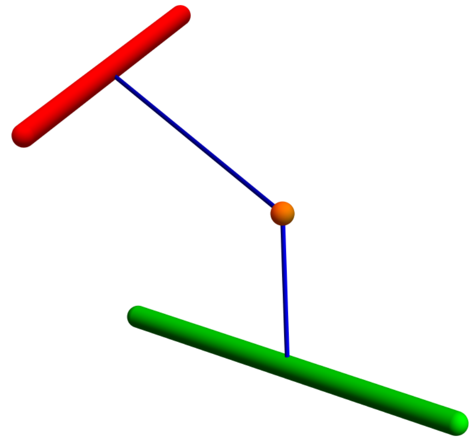
One miracle about the spiral is that arc length from 0 to t multiplied with curvature at t is constant. Jacob Bernoulli called it the curve the "Spira mirabilis" which means "miraculous spiral".

Problem 8) (10 points)

Given a line $\vec{r}(t) = [1+t, t, t]$ and a line $\vec{s}(t) = [1-t, 1+t, 1-t]$.

a) (6 points) Find the sum of the distances of the point $(0,0,0)$ to the two lines.

b) (4 points) Find the distance between the two lines.



Problem 9) (10 points)

Parametrize the following surfaces in space. As usual r, θ, z are cylindrical and ρ, θ, ϕ are spherical coordinate variables. You do not need to give bounds on the parameters.

a) (2 points) Parametrize $y = \cos(3x) - \sin(3z)$ as

$\vec{r}(x, z) =$

b) (2 points) Parametrize $\rho = 2 + \cos(8\theta + 5\phi)$ as

$\vec{r}(\theta, \phi) =$

c) (2 points) Parametrize $r^2 - z^2 = 1$ as

$\vec{r}(\theta, z) =$

d) (2 points) Parametrize $x = 0$ as

$\vec{r}(y, z) =$

e) Decide whether none, one, or both of the grid curves u = 1, v = 1 is a circle, if

$$\vec{r}(u, v) = [(3u + u\cos(v))\cos(2u), (3u + u\cos(v))\sin(2u), (3u + u\sin(v))]$$

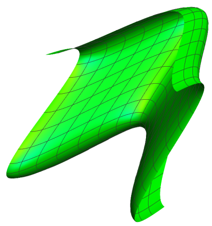
(1 point) Is the curve $u = 1$ a circle?

Yes or No

(1 point) Is the curve $v = 1$ a circle?

Yes or No

Illustrations: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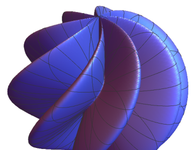
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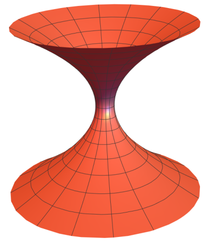
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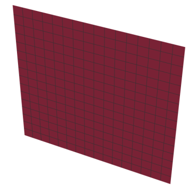
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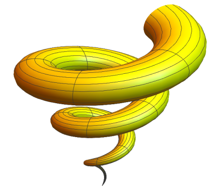
e)