

Name:

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MWF 1:30 Flor Orosz-Hunziker
MWF 3 Waqar Ali-Shah

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work.** Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) True/False (TF) questions (20 points)
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Mark for each of the 20 questions the correct letter. No justifications are needed.)

T	F
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For

- 2) 

T	F
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 There are unit vectors  $\vec{v}$  and  $\vec{w}$  in space for which  $|\vec{v} \times \vec{w}| = 2$ .
- 3) 

T	F
---	---

 The vector  $[4, 5, 0]$  is perpendicular to the plane  $-5x + 4y + z = 2$ .
- 4) 

T	F
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 The distance between the cylinders  $x^2 + z^2 = 1$  and  $x^2 + (z - 3)^2 = 1$  is 3.
- 5) 

T	F
---	---

 The vector projection of  $[2, 3, 1]$  onto  $[1, 1, 1]$  is parallel to  $[1, 1, 1]$ .
- 6) 

T	F
---	---

 The equation  $\rho \sin(\theta) \sin(\phi) = 2$  in spherical coordinates defines a plane.
- 7) 

T	F
---	---

 There is a closed planar curve for which the arc length is 1.
- 8) 

T	F
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 If we know the intersection of a graph  $z = f(x, y)$  with the coordinate planes  $x = 0, y = 0$  and  $z = 0$ , the function  $f$  is determined uniquely.
- 9) 

T	F
---	---

 The surface given in cylindrical coordinates as  $r = z^2$  is a paraboloid.
- 10) 

T	F
---	---

 For all vectors  $\vec{v}$  and  $\vec{w}$  we have  $\vec{w} \cdot (\vec{v} \times (\vec{v} \times \vec{w})) \leq 0$ .
- 11) 

T	F
---	---

 If  $\vec{u}, \vec{v}$ , and  $\vec{w}$  are unit vectors then the volume of the parallelepiped spanned by  $\vec{u}, \vec{v}$ , and  $\vec{w}$  is largest when the parallelepiped is a cube.
- 12) 

T	F
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 If a point is moving along a straight line parametrized by  $\vec{r}(t)$  then the velocity  $\vec{r}'(t)$  vector and acceleration vector  $\vec{r}''(t)$  must be parallel.
- 13) 

T	F
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 The parametrization  $\vec{r}(u, v) = [v \cos(u), v \sin(u), v]$  with  $0 \leq u < 2\pi$  and  $v \in \mathbf{R}$  is a cylinder.
- 14) 

T	F
---	---

 If two lines in space are not parallel, then they must intersect.
- 15) 

T	F
---	---

 If two planes do not intersect, then their normal vectors are parallel.
- 16) 

T	F
---	---

 $(\vec{i} \times \vec{j})$  and  $(\vec{i} \times (\vec{i} \times (\vec{i} \times \vec{j})))$  are parallel.
- 17) 

T	F
---	---

 The surface parametrized by  $\vec{r}(u, v) = [\sin(u) \sin(v), \sin(u) \cos(v), \cos(u)]$  with  $0 \leq v < 2\pi, 0 \leq u \leq \pi$  is a sphere.
- 18) 

T	F
---	---

 The unit tangent vector  $\vec{T}$  to a curve at a given point is independent of the parametrization up to a factor of  $-1$ .
- 19) 

T	F
---	---

 $z^2 = r^2(\cos^2(\theta) - \sin^2(\theta)) + 1$  is a one-sheeted hyperboloid.
- 20) 

T	F
---	---

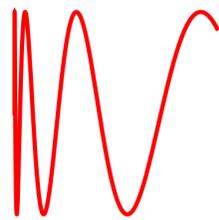
 If  $\vec{a} \cdot \vec{b} > 0$  and  $\vec{b} \cdot \vec{c} > 0$ , then  $\vec{a} \cdot \vec{c} > 0$ .

	Total
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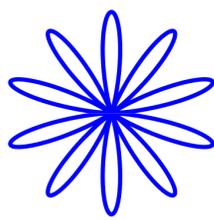
Problem 2) (10 points)

No explanations needed. I,II,III,O appear all once in each box.

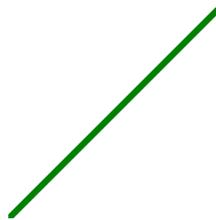
a) (2 points) Match curves with their parametrizations  $\vec{r}(t)$ . Enter O, if there is no match.



I



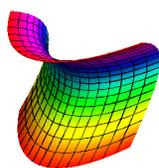
II



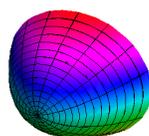
III

Parametrization $\vec{r}(t) =$	O, I,II,III
$\sin^2(5t)[\cos(t), \sin(t)]$	
$[t^3, \sin(7t)]$	
$[t^5, 1 + t^5]$	
$[\sin(t), \cos(t)]$	

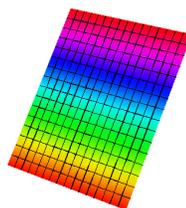
b) (2 points) Match the parametrization. Enter O, where no match.



I



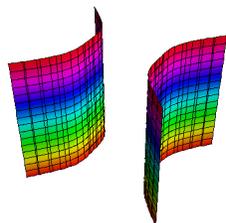
II



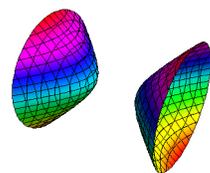
III

$\vec{r}(s, t)$	O,I,II,III
$[1 - t, 1 + s, 2 + s]$	
$[s, t^2 - s^2, t]$	
$[t \cos(s), t \sin(s), s]$	
$[s \cos(t), s^2, s \sin(t)]$	

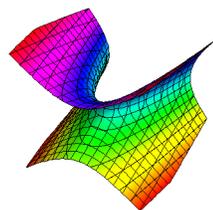
c) (2 points) The pictures show contour surfaces. Enter O, where no match.



I



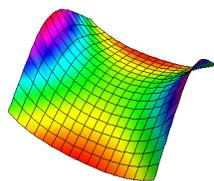
II



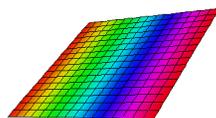
III

$g(x, y, z) =$	O, I,II,III
$x^2 - y^2 + z^2 = -1$	
$x^2 - y^2 = 1$	
$x^4 + z = 1$	
$x^2 + y - z^2 = 1$	

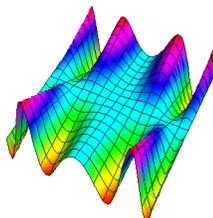
d) (2 points) Match the graphs  $z = f(x, y)$  with the functions. Enter O, where no match.



I



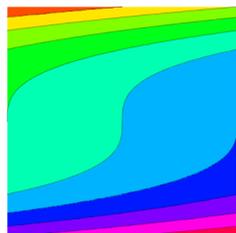
II



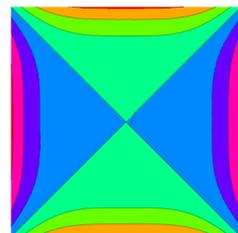
III

function $f(x, y) =$	O,I,II,III
$2x$	
$e^{-2x^2-2y^2}$	
$e^{x^2-y^2}$	
$y \sin(x^2)$	

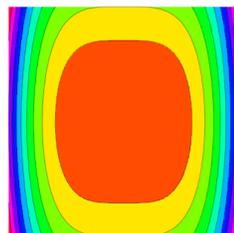
e) (2 points) Match the family of level curves with  $f(x, y)$ . Enter O, where no match.



I



II



III

Function $f(x, y) =$	O,I,II,III
$x^4 + y^2$	
$x^4 - y^4$	
$x - y$	
$x - y^3$	

Problem 3) (10 points)
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No explanations needed. In 3a), in each row check only one box.

a) (4 points) The intersection of a plane with a cone  $S : x^2 + y^2 - z^2 = 0$  is called a **conic section**. What curve do we get?

Intersect $S$ with	hyperbola	parabola	circle	line
$z = 1$ gives a				
$z = x$ gives a				
$z = x + 1$ gives a				
$x = 1$ gives a				

b) (3 points) By intersecting the upper half sphere  $x^2 + y^2 + z^2 = 5, z > 0$  with the hyperboloid  $x^2 + y^2 - z^2 = -3$  we get a curve. Which one? Check exactly one box.

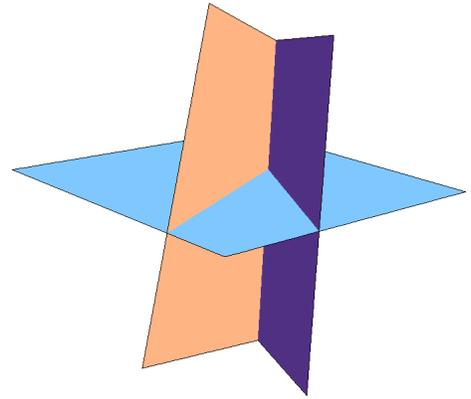
$\vec{r}(t) = [\cos(t), \sin(t), 2]$	
$\vec{r}(t) = [0, 0, t]$	
$\vec{r}(t) = [\cos(t), \sin(t), 2t]$	

c) (3 points) Which of the following surface parametrizations gives a one sheeted hyperboloid? Check exactly one box.

$\vec{r}(t, s) = [s, t, s^2 - t^2]$	
$\vec{r}(t, s) = [\sqrt{1 + s^2} \cos(t), \sqrt{1 + s^2} \sin(t), s]$	
$\vec{r}(t, s) = [\sqrt{1 - s^2} \cos(t), \sqrt{1 - s^2} \sin(t), s]$	

Problem 4) (10 points)
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We are given two planes  $x + y + z = 1$  and  $x - y - z = 2$ . Find a third plane which contains the point  $(1, 0, 0)$  and which is perpendicular to both.

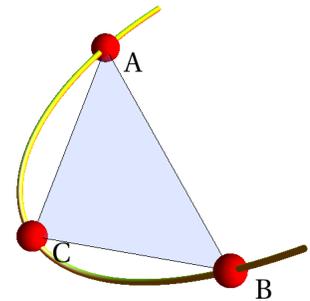


Problem 5) (10 points)

We are given a curve  $\vec{r}(t) = [1 + t, t^2, t^3]$ .

a) (5 points) Find the area of the triangle with vertices  $A = \vec{r}(-1)$ ,  $B = \vec{r}(1)$  and  $C = \vec{r}(0)$ .

b) (5 points) Find an equation  $ax + by + cz = d$  for the plane through  $A, B, C$ .

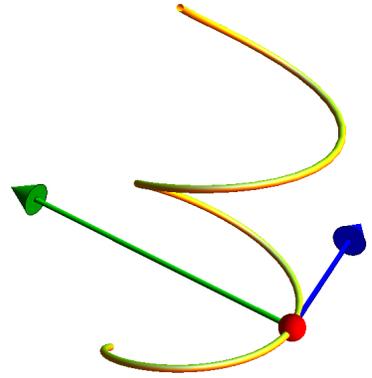


Problem 6) (10 points)

- a) (3 points) Find the unit tangent vector  $\vec{T}(t)$  of the curve  $\vec{r}(t) = [t^2, \cos(t^2\pi), \sin(t^2\pi)]$  at  $t = 1$ .
- b) (3 points) What is the acceleration vector  $\vec{r}''(t)$  at  $t = 1$ ?
- c) (4 points) Use the formula

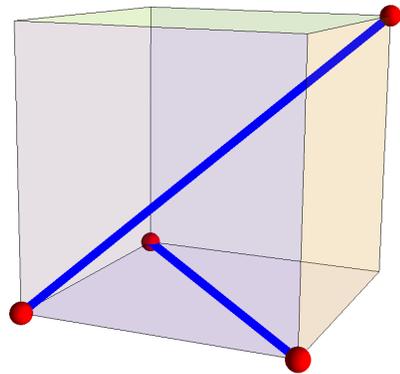
$$\kappa(t) = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3}.$$

to compute  $\kappa(1)$ .



Problem 7) (10 points)

What's the closest that the long diagonal of the unit cube connecting the corners  $(0, 0, 0)$  to  $(1, 1, 1)$ , comes to the diagonal of a face connecting the corners  $(1, 0, 0)$  and  $(0, 1, 0)$ ?



Problem 8) (10 points)

In a parallel universe of ours, the inhabitants live under a “Newton’s law” of gravity in which the “jerk”  $\vec{r}'''(t)$  rather than the acceleration is constant. Suppose that  $\vec{r}'''(t) = [0, 0, -10]$  for all  $t$ .

- a) (3 points) Find  $\vec{r}''(t)$  if you know  $\vec{r}''(0) = [0, 0, 0]$ .
- b) (3 points) Now find  $\vec{r}'(t)$  if we know also  $\vec{r}'(0) = [1, 0, 0]$ .
- c) (4 points) Finally find  $\vec{r}(t)$  if we know additionally  $\vec{r}(0) = [0, 0, 10]$ .



Problem 9) (10 points)

a) (2 points) A fly is trapped inside a unit cubicle made of planar glass panes. It flies, starting at  $t = 0$  at the origin  $(0, 0, 0)$  along the curve

$$\vec{r}(t) = \left[ t, \frac{t^2}{\sqrt{2}}, \frac{t^3}{3} \right].$$

At what time does it bump into the glass wall  $x = 1$ ?

b) (4 points) Find the impact angle (= the angle between the normal vector of the plane and the velocity vector).

c) (4 points) How long is the path it has followed from  $t = 0$  to the impact point?

