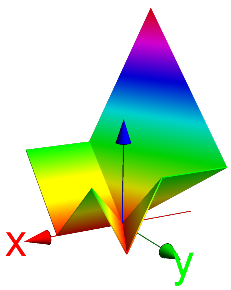
Homework 4: Functions

This homework is due Wednesday, 9/18/2019.

1 Match the following graphs with the functions $f(x, y)$.

1



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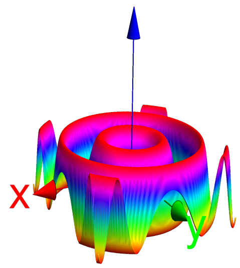
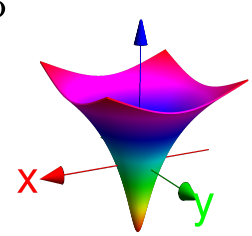


3



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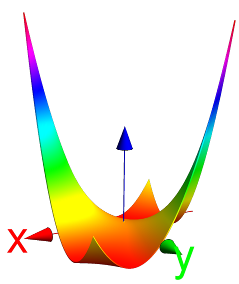

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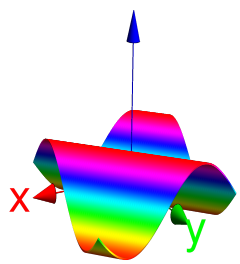

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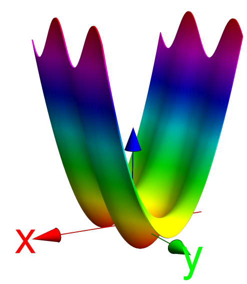
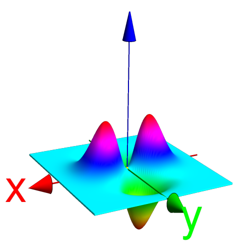


7



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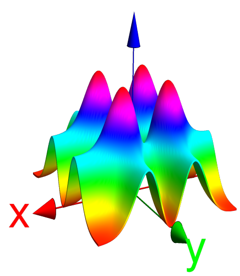
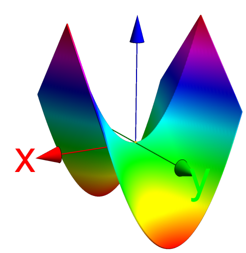


9



$f(x,y) =$	1-9	$f(x,y) =$	1-9	$f(x,y) =$	1-9
$e^{-x^2-y^2}(x^2-y^2)$		x^2y^2-7xy		$e^{-x^2}x^2+e^{-y^2}y^2$	
$\sin(x^2+y^2)$		$\|x\|-\|y-\|x\|\|\|$		$x^2-\|y\|$	
$\log(x^2+y^2+1)-1$		$\sin(x+y)$		$x^2-\sin(2y)$	

Solution:

Left row: 7,2,3, Middle row: 4,1,5 right row: 8,9,6

2 Try to solve this problem without technology (if you must, do so):
a) Draw the contour map of $f(x, y) = 1 + x^2 + y^2 + e^{-(x^2+y^2)}$.
b) What is the domain of the function $f(x, y) = \sqrt{x^2y - y^2x}$?
c) Make a picture of the graph of $f(x, y) = 2|x| + 3|y| - 6$.

Solution:
a) Note that this is a function of $x^2 + y^2$ so that the level curves are circles. b) Look at the places where the function inside the square root is positive. Draw the curves $f(x, y) = 0$ which are three lines. c) In every quadrant we have a plane. These are four planes coming together a tone point. It looks like an upside down pyramid if you cut it at z=0.

3 In this problem you as usual have the option of using technology.
a) Plot the graph and contour map of $f(x, y) = \frac{x^2-y^2}{x^2+y^2}$
b) Plot the graph and contour map of $f(x, y) = \frac{xy}{x^4+y^4}$.

Solution:
a) The contours are lines through the origin.
b) This needs computer assistance. We see a Klover type surface.

4 Find an equation for the surface consisting of all points P for which the distance from P to the x-axes is twice the distance from P to the yz-plane. Identify the surface.

Solution:

The distance from a point to the x-axis is $\sqrt{y^2+z^2}$. The distance to the yz-plane is $|x|$. The surface consisting of all these points is defined by $\sqrt{y^2+z^2} = 2|x|$. This is a perfectly acceptable answer, but simplifying is nicer: square both sides and we get $y^2 + z^2 = 4x^2$. From this equation, we can determine that this is a cone whose axis is the x-axis.

5 For the following, solve without technology.

a) Draw the surface $4y^2 + z^2 - x - 16y - 4z + 20 = 0$.

b) Draw the surface $x - z^2 + y^2 + 4y = 1$.

Solution:

(a) Rearranging this equation and completing the square yields:

$$\begin{aligned} 4y^2 - 16y + z^2 - 4z &= x - 20 \\ 4(y^2 - 4y + 4) + z^2 - 4z + 4 &= x - 20 + 16 + 4 \\ 4(y-2)^2 + (z-2)^2 &= x \end{aligned}$$

We can see that this is the equation of an elliptic paraboloid with vertex (0, 2, 2) that opens in the positive-x direction along the axis $y = 2, z = 2$. The paraboloid is stretched in the y direction by a factor of $\frac{1}{2}$. (b) Rearranging this equation and completing the square yields:

$$\begin{aligned} x - z^2 + y^2 + 4y + 4 &= 5 \\ x - z^2 + (y+2)^2 &= 5 \\ x &= z^2 - (y+2)^2 + 5 \end{aligned}$$

We can see that this equation describes a hyperbolic paraboloid (i.e. a saddle). The saddle is centered at (0, -2, 0).

Main definitions

The **domain** D of a function $f(x, y)$ is the set of points where f is defined, the **range** is $\{f(x, y) \mid (x, y) \in D\}$. The **graph** of $f(x, y)$ is the surface $\{(x, y, f(x, y)) \mid (x, y) \in D\}$ in $\mathbb{R}^3$. The set $f(x, y) = c = \text{const}$ is **contour curve** or **level curve** of f. The collection of contour curves $\{f(x, y) = c\}$ is called the **contour map** of f. A function of three variables $g(x, y, z)$ can be visualized by **contour surfaces** $g(x, y, z) = c$, where c is constant. **Traces**, the intersections of the surfaces with the coordinate planes help to draw them. Examples: the elliptic paraboloid $z - x^2 - y^2 = 0$ and hyperbolic paraboloid $z - x^2 + y^2 = 0$ are examples of graphs $z - f(x, y) = 0$. The one sheeted hyperboloid $x^2 + y^2 - z^2 = 1$ and two sheeted hyperboloid $x^2 + y^2 - z^2 = -1$ or cylinder $x^2 + y^2 = 1$ are examples of surfaces of revolution $x^2 + y^2 - g(z) = 0$. The ellipsoid $x^2/4 + y^2/9 + z^2/16 = 1$ has ellipses as traces.