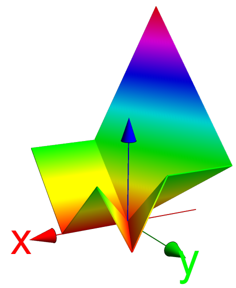
Homework 4: Functions

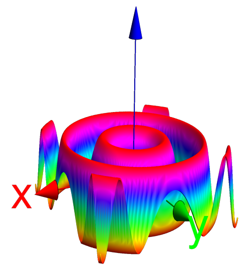
This homework is due Wednesday, 9/18/2019.

1 Match the following graphs with the functions $f(x,y)$.

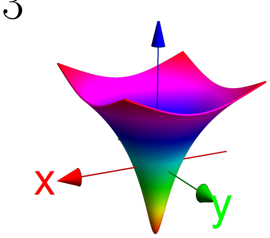
1



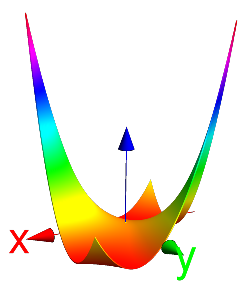
2



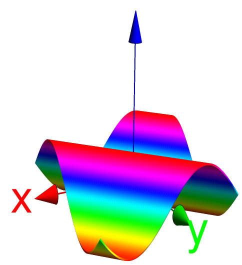
3



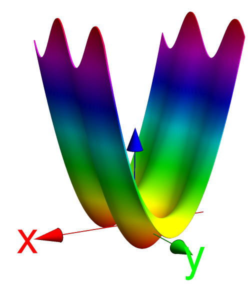
4



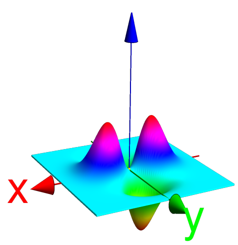
5



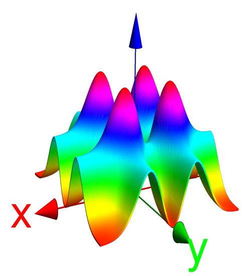
6



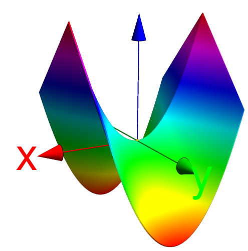
7



8



9



$f(x,y) =$	1-9	$f(x,y) =$	1-9	$f(x,y) =$	1-9
$e^{-x^2-y^2}(x^2-y^2)$		x^2y^2-7xy		$e^{-x^2}x^2+e^{-y^2}y^2$	
$\sin(x^2+y^2)$		$\lvert x-\lvert y-\lvert x\rvert\rvert\rvert$		$x^2-\lvert y\rvert$	
$\log(x^2+y^2+1)-1$		$\sin(x+y)$		$x^2-\sin(2y)$	

2 Try to solve this problem without technology (if you must, do so):

a) Draw the contour map of $f(x,y) = 1 + x^2 + y^2 + e^{-(x^2+y^2)}$.

b) What is the domain of the function $f(x,y) = \sqrt{x^2y - y^2x}$?

c) Make a picture of the graph of $f(x,y) = 2|x| + 3|y| - 6$.

3 In this problem you as usual have the option of using technology.
a) Plot the graph and contour map of $f(x,y) = \frac{x^2-y^2}{x^2+y^2}$
b) Plot the graph and contour map of $f(x,y) = \frac{xy}{x^4+y^4}$.

4 Find an equation for the surface consisting of all points P for which the distance from P to the x-axes is twice the distance from P to the yz-plane. Identify the surface.

5 For the following, solve without technology.
a) Draw the surface $4y^2 + z^2 - x - 16y - 4z + 20 = 0$.
b) Draw the surface $x - z^2 + y^2 + 4y = 1$.

Main definitions

The **domain** D of a function $f(x,y)$ is the set of points where f is defined, the **range** is $\{f(x,y) \mid (x,y) \in D\}$. The **graph** of $f(x,y)$ is the surface $\{(x,y,f(x,y)) \mid (x,y) \in D\}$ in $\mathbb{R}^3$. The set $f(x,y) = c = \text{const}$ is **contour curve** or **level curve** of f. The collection of contour curves $\{f(x,y) = c\}$ is called the **contour map** of f. A function of three variables $g(x,y,z)$ can be visualized by **contour surfaces** $g(x,y,z) = c$, where c is constant. **Traces**, the intersections of the surfaces with the coordinate planes help to draw them. Examples: the elliptic paraboloid $z - x^2 - y^2 = 0$ and hyperbolic paraboloid $z - x^2 + y^2 = 0$ are examples of graphs $z - f(x,y) = 0$. The one sheeted hyperboloid $x^2 + y^2 - z^2 = 1$ and two sheeted hyperboloid $x^2 + y^2 - z^2 = -1$ or cylinder $x^2 + y^2 = 1$ are examples of surfaces of revolution $x^2 + y^2 - g(z) = 0$. The ellipsoid $x^2/4 + y^2/9 + z^2/16 = 1$ has ellipses as traces.