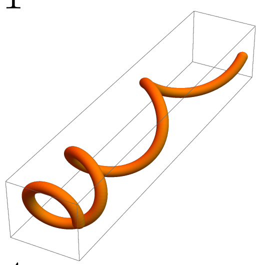
Homework 5: Parametrized curves

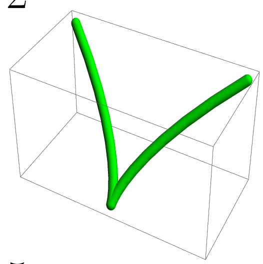
This homework is due Friday, 9/20/2019.

1 Match the curves:

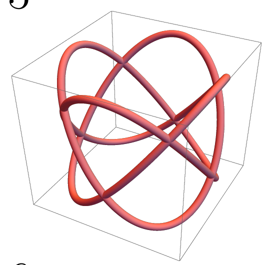
1



2



3



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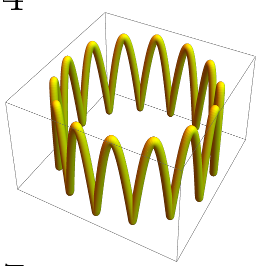

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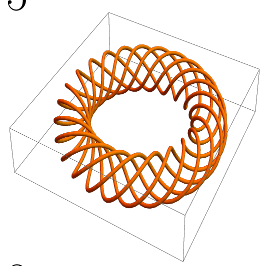

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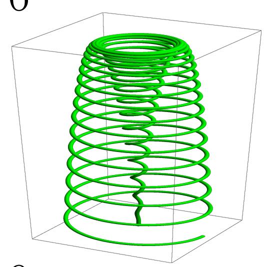

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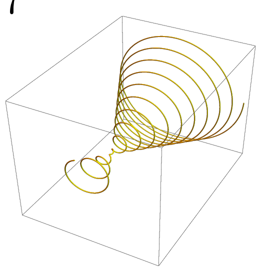

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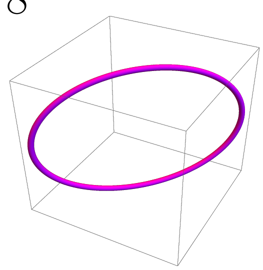
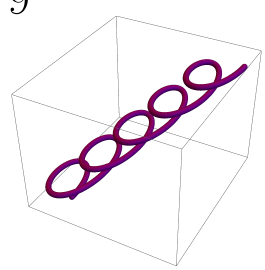


9



$\vec{r}(t) =$	1-9
$[\cos(t), \sin(t), \lvert\cos(7t)\rvert]$	
$[t + \cos(5t), t + \sin(5t), t]$	
$[(4 + \cos(24t))\cos(5t), (4 + \cos(24t))\sin(5t), \sin 24t]$	
$[t^3, t^2, t^2]$	
$[t\cos(8t), t\sin(8t), t(8\pi - t)]$	
$[t\cos(10t), 2t, t\sin(10t)]$	
$[6 + \cos(3t), \sin(3t), \cos(4t)]$	
$[\cos(t), \cos(t), \sin(t)]$	
$[\cos(t), t^2, \sin(t)]$	

2 Parametrize the intersection of the wave surface $z = \sin(x)+\sin(y)$ with the elliptic cylinder $(x + 5)^2/9 + (y - 1)^2/16 = 1$.

3 a) Two particles travel along the space curves $\vec{r}_1(t) = [t, t^2, t^3]$ and $\vec{r}_2(t) = [1 + 2t, 1 + 6t, 1 + 14t]$. Do the particles collide? Do the particle paths intersect?

b) If $\vec{r}(t) = [\cos(t), 2\sin(t), 4t]$, find $\vec{r}\,'(0)$ and $\vec{r}\,''(0)$. Then compute $|\vec{r}\,'(0) \times \vec{r}\,''(0)|/|\vec{r}\,'(0)|^3$. One calls this the curvature.

4 Find the point of intersection of two tangent lines to the curve $\vec{r}(t) = [\sin(\pi t), 2\sin(\pi t), \cos(\pi t)]$. The first tangent is at $t = 0$, the second at $t = 0.5$.

5 A particle moving along a curve $\vec{r}(t)$ has the property that $\vec{r}\,''(t) = [5, 6t, 8 + 4\sin(2t)]$. We know $\vec{r}(0) = [8, 1, 2]$ and $\vec{r}\,'(0) = [7, 0, 3]$. What is $\vec{r}(\pi)$?

Main definitions

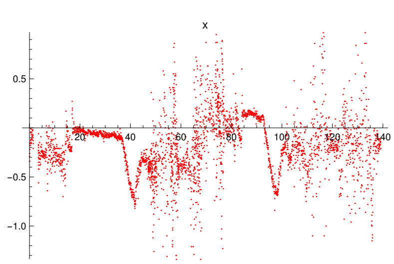
The parametrization of a space curve is $\vec{r}(t) = [x(t), y(t), z(t)]$. The **image** of r is a **parametrized curve** in space. If $\vec{r}(t) = [x(t), y(t), z(t)]$ is a curve, then $\vec{r}\,'(t) = [x'(t), y'(t), z'(t)]$ is called the **velocity** at time t. Its length $|\vec{r}\,'(t)|$ is called **speed** and $\vec{T}(t) = \vec{r}\,'(t)/|\vec{r}\,'(t)|$ is called **unit tangent vector** or direction of motion. The vector $\vec{r}\,''(t)$ is called the **acceleration**. When knowing the acceleration and $\vec{r}\,'(0)$ and $\vec{r}(0)$ we can get back position $\vec{r}(t)$ by integration. Similarly, if we know $\vec{r}\,''(t)$ at all times and $\vec{r}(0)$ and $\vec{r}\,'(0)$, we can compute $\vec{r}(t)$ by integration.

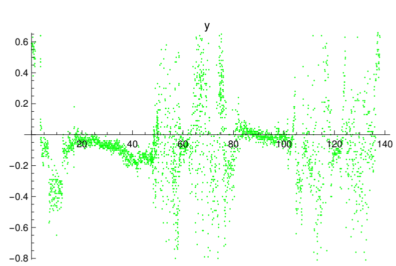
Data visualization

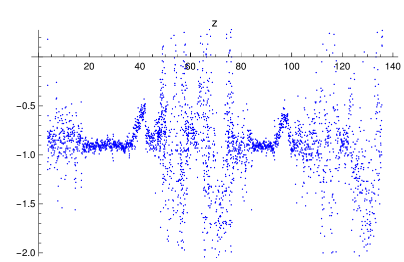
This is an extra credit problem.

In the blog *http : //blog.robindeits.com/2013/11/11/roller−coaster−visualizations/*, Robin Deits uses a smart phone to measure the accelerations during a roller coaster ride on **Cedar point**, the foremost American roller coaster park. Robin made the data available on a github repository mentioned in that blog.

The following three pictures show the **x-acceleration** $x''(t)$, the **y-acceleration** $y''(t)$ and the **z-acceleration** data $z''(t)$ plotted for the mine ride roller coaster. These data are not given as functions but as **data points** as the smart phone has measured the data at a discrete set of points.







Our goal is to find a way to visualize the actual shape $\vec{r}(t) = [x(t), y(t), z(t)]$ of the roller coaster from acceleration data $\vec{r}\,''(t)$. By summing up the acceleration data, and then sum up the data again, we can reconstruct the position $\vec{r}(t)$.

On the course website http://sites.fas.harvard.edu/˜math21a/data/rollercoaster.nb, you have a notebook "rollercoaster.nb" which allows you to experiment. Included there are a few lines of code which reconstruct the curve from the data of Robin.

a) To get extra credit, please download Mathematica and run the notebook which is provided. There is a parametrization $\vec{r}(t)$ given there for which the $\vec{r}''(t)$ is evaluated at $M = 200$ points. Print out the plot of the reconstructred data points. (Just evaluate the code).

b) By thinking about Riemann sums, discuss why the reconstructed data points do not match exactly the data points of the curve. What happens when M gets larger? Try out, what happens if you use larger M.

c) In the second part of the notebook, we reconstruct the roller coaster shape from data which were measured by Robin's smartphone. Please run the notebook and plot out this curve.