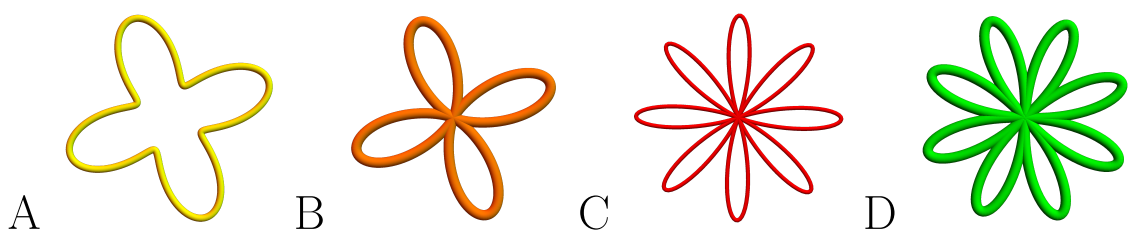
# Homework 7: Polar coordinates

This homework is due Wednesday, 9/25.

1 a) Change to polar coordinates: $(x, y) = (2, -2\sqrt{3})$.
b) Change to Cartesian coordinates: $(r, \theta) = (3, \pi/2)$
c) Change to cylindrical coordinates: $(x, y, z) = (-3\sqrt{3}, 3, 27)$
d) Change to Cartesian coordinates $(r, \theta, z) = (7, 3\pi/4, -11)$

**Solution:**
a) $(r, \theta) = (4, -\pi/3)$.
b) $(x, y) = (0, 3)$
c) $5\pi/6$
d) $(7/\sqrt{2}, 7/\sqrt{2}, -11)$.

2 Match the following curves with the formula giving them in polar coordinates




| Formula | Enter A-D |
|---|---|
| $r = \lvert\sin(4\theta)\rvert$ | |
| $r = 2 + \sin(4\theta)$ | |
| $r = \cos^2(4\theta)$ | |
| $r = 1 + \sin(4\theta)$ | |

3 a) The polar curve $r = \lvert\sin(21\theta)\rvert$ is called a rose. How many petals does it have?

b) Identity the surface given in cylindrical coordinates as $r^2 = 1 + (z-5)^2$.
c) Write the hyperboloid $x^2 + y^2 - (z-1)^2 = 4$ in cylindrical coordinates.
d) Identify the surface $2r^2 - z = -1$.

**Solution:**
a) 42. b) It is a one sheeted hyperboloid. c) It is $r^2 - *z - 1)^2 - 4$.
d) It is a paraboloid.

4 a) Identity the surface whose equation is given in cylindrical coordinates as

$$\cos(\theta) + \sin(\theta) = 1/r \ .$$

b) Write the ellipsoid $x^2/4 + y^2/9 + z^2/16 = 1$ in cylindrical coordinates.

**Solution:**
a) It is a plane: if we multiply both sides by $r$, we get the equation $r\cos\theta + r\sin\theta = 1$, i.e $x + y = 1$.
b) $r^2\cos^2(\theta)/4 + r^2\sin^2(\theta)/9 + z^2/16 = 1$.

5 Use the Mathematica command "RevolutionPlot3D" to plot the surface which is given by

$$z = (\sin(7\theta) + \cos(7\theta))e^{-r^2} \ .$$

(See example Mathematica command below. As usual, you can print the result out and include in the homework.)

Example:

```
RevolutionPlot3D[t, {r, 0, 1}, {t, 0, 2 Pi}]
```

## Main definitions

A point $(x, y)$ in the plane has **polar coordinates** $r = \sqrt{x^2 + y^2} \geq 0, \theta = \arctan(y/x)$. Important is the relation $(x, y) = (r\cos(\theta), r\sin(\theta))$. To get the angle, it is custom to chose arctan values in $(-\pi/2, \pi/2]$ and add $\pi$ if $x < 0$ or $x = 0, y < 0$. This gives a value in $\theta \in [0, 2\pi)$. A point $(x, y, z)$ in space has the **cylindrical coordinates** $(r, \theta, z)$, where $(r, \theta)$ are the polar coordinates of $(x, y)$. A curve given in polar coordinates as $r(\theta) = f(\theta)$ is called a **polar curve**. It can in Cartesian coordinates be described as $\vec{r}(t) = [f(t)\cos(t), f(t)\sin(t)]$.